A Model of Dynamic Information Disclosure

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Abstract

We study a dynamic game in which a financial expert seeks to optimize the utilization of her private information either by information disclosure to an investor or by self-use. The investor may be aligned or biased: an aligned investor always cooperates on the disclosed information, whereas a biased investor may strategically betray the expert. We characterize the joint dynamics of the expert's information disclosure and the investor's type revelation and show that, by a process of gradual information disclosure, the expert can significantly alleviate the hold-up effect exerted by the biased investor. In particular, we show that the equilibrium dynamics of the players' interactions is unique. We also examine how the expert can further improve her utilization of information by committing to a deadline or by committing to a particular pattern of information disclosure.

Key Words: Private information; Information disclosure; Hold-up effect; Gradualism; Commitment.

JEL Classification: C73, D82, D83.

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1. Introduction

As Hayek (1945) claimed more than half a century ago, the central issue in a variety of economic and social interactions is how to utilize information efficiently.[1] Theoretical and practical developments have contributed many effective patterns to achieve the efficiency of information utilization, for instance, the widespread usages of patent protection, contractual enforcement and property rights allocation. However, when these tools are unavailable, as is often the case, a hold-up problem may arise and thereby discourage the utilization of information: once an initially uninformed party has learned about the valuable information from another party, his incentive to pay for the information weakens as now himself is informed (Arrow, 1962). We provide an equilibrium analysis of information utilization in this paper and address how a process of gradual information disclosure helps to alleviate the hold-up problem.

To gain better understanding of this study, consider the situation that a financial expert, who knows some investment opportunities, seeks cooperation from a fund manager to optimize the utilization of her information. Being aware that the fund manager could be motivated to seize all the information value instead of to establish a cooperative relationship, the financial expert may strategically slow down the release of her information to reduce the risk of being exploited. As the uncertainty about the fund manager's motives is gradually resolved, the financial expert eventually becomes confident enough to release all her information.

For another example, consider the situation that an international auto company aims to open the market of a developing country through technology cooperation with a local company. Lack of a well-established law system in this country makes the auto company's technology transfer under the risk of being leaked or stolen. As a response, the auto company may optimally choose to transfer some outdated technology first, which gives it the chance to learn about its partner. Contingent on the local company's behaviors, the auto company can further determine to transfer newer technology or exit the market.

These examples share some similarities. First, information is divisible and is transmitted or disclosed gradually, which allows the values of different parts of information to be realized independently. For instance, even the outdated technology generates revenues to the auto company. Second, contractual enforcement on information disclosure may be incredible or even absent, which gives rise to the potential hold-up problem. As a result, the parties' interactions need to be self-enforcing. Finally, timing cost could be an important factor affecting the utilization of

[1] See Hayek (1945), page 519-520: "The economic problem of society is thus not merely a problem of how to allocate "given" resources..., it is a problem of the utilization of knowledge not given to anyone in its totality."

information. For instance, in a volatile stock market investment opportunities lose their values rapidly over time, whereas delaying the transfer of the newest technology could cause the auto company to lose market shares to its competitors.

Taking the first example as our leading example in this paper, we develop a dynamic game that examines the gradualism of information disclosure. A financial expert is endowed with an amount of private information that is valuable in the stock market, but she can only utilize it inefficiently on her own because of her limited access to the market. On the other hand, an investor has the potential to maximize the value of the expert's information, but he is initially in lack of the relevant information. In consequence, efficient utilization of information requests information disclosure between the two players. Interactions go as follows. In each period, if the expert chooses to use some information by herself, then the investor is inactive; if the expert discloses some information to the investor, then the latter can either cooperate, which is mutually beneficial, or betray, which benefits himself solely. The investor may be aligned or biased. An aligned investor always cooperates whereas a biased investor may strategically betray. The expert is initially uncertain about the investor's type and thereby she has to learn about it over time. Given the discounting cost, the expert's goal is to optimize the payoff from her information when external contracts are infeasible.

From the expert's perspective, for any amount of her information, information disclosure to the investor dominates self-use of information if the investor's cooperation can be induced, but self-use is better than information disclosure if the investor intends to betray. As a result, the expert faces two main trade-offs in determining her utilization of information. The first is whether information should be disclosed. Although self-use of information gives rise to a substantial efficiency loss, the timing cost and the rents captured by the biased investor need to be considered when information is disclosed. The second is, if information is disclosed, how fast the process of disclosure should be. A longer process is more timing costly but it leaves less information rents to the biased investor, whereas a shorter process saves timing cost but should give more rents to the biased investor.

We construct an equilibrium in which the expert's trade-offs are resolved by a finite sequence of cut-off values, which are the expert's beliefs about the investor's type. If the investor is highly aligned, the probability for the expert to be betrayed by the investor is relatively low and it is optimal for her to disclose information quickly. If the investor is moderately aligned, it is better for the expert to slow down the process of information disclosure to weaken the biased investor's incentive on betrayal. Finally, if the investor is sufficiently biased, any process of disclosure becomes too costly and the expert optimally chooses to self-use her information. This characterization gives an explicit insight about how the expert can benefit from a gradual disclosure of her information.

Moreover, we show that the equilibrium of this game is essentially unique. The critical determinant of the equilibrium uniqueness is that the completion of the process of information disclosure is endogenously determined. Specifically, in a particular period if the biased investor betrays with a probability larger than the one played on the equilibrium path, after observing cooperation the expert believes that the investor is more aligned and thereby prefers to speed up her information disclosure in the continuation game, but then the biased investor actually should cooperate for sure in the current period. Conversely, if in such a period the biased investor betrays with a probability less than the one played on the equilibrium path, after observing cooperation the expert is still very cautious about the investor and thereby prefers to slow down her information disclosure in the continuation game, which implies that the biased investor should betray for sure in this period. As a result, for any amount of information disclosure by the expert the biased investor's response is unique, and the expert's problem is much like a decision problem that she chooses the optimal plan of information disclosure from all the feasible plans, which is also unique.

In many circumstances the time period for information disclosure is limited. We examine how the existence of a deadline affects the expert's information utilization and, specifically, how the expert's payoff is improved if she can commit to a deadline. In case the deadline period is reached, the expert's choice is restricted in a way that no gradual disclosure of information, thereby no gradual learning about the investor's type, is allowed in the future. Such a restriction lowers the expert's *ex post* payoff. However, expecting that the expert is more willing to disclose all her remaining information in the deadline period even her posterior belief is not sufficiently high, the biased investor can effectively reduce his betrayal probabilities in the periods before the deadline, which increases the expert's *ex ante* payoff. We show that for moderate initial beliefs the expert's equilibrium payoff is strictly improved if she commits to a proper deadline.

We also examine the effects on the expert's information utilization if she can fully commit to a particular process of information disclosure. In equilibrium, the optimal process with commitment has a property that the biased investor is induced to cooperate in all periods except the last one. In other words, the amount of information disclosed in the final period serves as a promised gift to the biased investor in rewarding his cooperation before this period, and the expert's problem in determining the optimal process is to trade off between the scale of the gift and the timing cost to deliver it. Not surprisingly, by committing to a process properly, the expert's payoff can be improved.

Besides the examples aforementioned, this game is also applicable to many other real-world situations. For example, if the valuable information refers to research ideas, then the game can address relationship building and termination between scientists. More broadly, if what the expert possesses is some sort of valuable assets, the game can be interpreted as a contribution game in which one party contributes inputs and the other one contributes productivity.

On a technical level, we deal with a game in which the action space (the amount of information to be utilized) varies over time. As a result, part of our contributions lies in the detailed construction of the equilibrium and the verification of the equilibrium uniqueness, which offers some novel insights to the study of games with similar technical properties.

Our paper is most closely related to Baliga and Ely (2010) and Horner and Skrzypacz (2011). Baliga and Ely (2010) consider a model in which a principal uses torture to extract information from an agent who may or may not be informed. In equilibrium, the informed agent initially resists but eventually concedes, and his information is gradually extracted. The equilibrium rate of information extraction is determined by how severe is the torture cost that the principal can exert on the agent. Horner and Skrzypacz (2011) develop a model in which an agent knowing a valuable state of nature can gradually reveal this state to a firm in exchange for payments. They address the equilibrium that maximizes the agent's *ex ante* incentives to learn about the state of nature and show that, in such an equilibrium, revealing information gradually strictly increases the agent's payoff and the process of information revelation always exhausts all the time periods. Our study shares the similarity with these papers by showing the qualitative result that gradual information disclosure could be beneficial to an informed player, but the underlying setup is quite different. Specifically, while discounting cost, outside option with self-use of information and endogenous ending of information disclosure are crucial to our findings, they have no roles in these papers. These differences enable us to offer new insights to many real world situations.

Gradualism also appears as the means to alleviate the hold-up effects in the literature on contribution games, for example, Admati and Perry (1991), Gale (2001), Lockwood and Thomas (2002), Marx and Matthews (2002) and Compte and Jehiel (2004). A key feature in our work is that gradual information disclosure arises due to asymmetric information about the investor's type, which is absent in those papers. Watson (1999, 2002) studies a contribution game with two-sided incomplete information and shows that the relationship between partners generally starts small and grows over time. In his papers, the level function along the time horizon is pre-determined before the game starts. As a result, at any instant time the players' actions are binary, either follow the level function or betray. In contrast, in our paper the expert's action space on information disclosure is a continuum in each period and the disclosure levels are determined along the process of play.

The gradual revelation of the investor's type is analytically related to the literature on reputation games, for example, Kreps and Wilson (1982), Milgrom and Roberts (1982) and Fudenberg and Levine (1989, 1992), and the literature on war of attrition with incomplete information, for example, Abreu and Gul (2000) and Damiano, Li and Suen (2012). A prominent difference is that, whereas in these papers the stage game is repeated and the only variables changing over time are the beliefs about the informed players' types, in our paper both the belief and the stage game vary over time as the amount of information remaining is decreasing.

Anton and Yao (1994) show that an inventor can appropriate a sizable share of an idea's market value from a buyer with the threat to reveal the idea to a competitor if the buyer defaults. Alternatively, Anton and Yao (2002) show that a seller can use partial disclosure to signal the full value of an idea and benefit from the buyers' competition in this idea. These papers allow enforceable contracts, but the timing structure of information disclosure is pre-determined. In contrast, in our work we focus on the endogenous timing structure of information disclosure instead of any explicit contracts.

2. The model

We consider a dynamic game involving two players: a financial expert (she or E) and an investor (he or I). At the beginning of the game the expert is endowed with an amount $Y_0 > 0$ of information, which refers to some investment opportunities that can be exploited in the stock market. A key feature regarding this amount of information is that, although the number Y_0 is common knowledge between the players, the detailed contents of the information is initially known only to the expert. Thus, for the investor being able to take actions with the information, he needs to learn about the relevant contents from the expert first. For simplicity, we assume that the expert's information is perfectly divisible.[2] Time is discrete and goes to infinite. Both players are risk-neutral and share a common discount factor $\delta \in (0, 1)$. A potential explanation for the factor δ is that information in the stock market loses its value over time if it is not utilized immediately.

Actions and payoffs are as follows. In period t, if the amount of remaining information is $Y_t > 0$ and the relationship between the players is still ongoing, the expert can either use an amount $x \leq Y_t$ by herself or disclose an amount $y \leq Y_t$ to the investor.[3] If an amount x is self-used, in this period

[2] This divisibility of information differentiates our study from most of the existing papers, for example, Fudenberg and Levine (1989, 1992), Abreu and Gul (2000) and Damiano, Li and Suen (2012), in which private information takes forms of nature states or types and it is intrinsically indivisible. In these papers dynamic information disclosure refers to a sequence of *probabilities* that a state or a type is gradually revealed, whereas in our paper, as well as in Baliga and Ely (2010), dynamic information disclosure refers to a sequence of *amounts* that information is gradually revealed, which will be much clearer in the following analysis.

[3] Self-use of information serves as an outside option to the expert and in the equilibrium we show later, whenever the expert self-uses her information, she self-uses all of it. Allowing the expert to disclose and self-use information

the expert and the investor's payoffs are x and 0 respectively. After the realization of payoffs, the game extends to the next period with remaining information $Y_{t+1} = Y_t - x$. On the other hand, if an amount y is disclosed, then the investor can choose to cooperate or betray. Cooperation generates a "success" and gives payoffs $\alpha_E y$ and $\alpha_I y$ to the expert and the investor respectively, whereas betrayal results in a "failure" and gives payoffs 0 and $\beta_I y$ to the expert and the investor. After the realization of payoffs, the game extends to period $t + 1$ with information $Y_{t+1} = Y_t - y$. The parameters satisfy

$$\alpha_E > 1, \quad \beta_I > \alpha_I > 0, \quad \text{and} \quad \alpha_E + \alpha_I \geq \beta_I,$$

which indicate that, while the investor's cooperation is both socially efficient and preferred to self-use by the expert, the investor can benefit more from his betrayal. This tension is the driving force underlying the players' interactions. In addition, we assume that the relationship is terminated whenever the investor betrays.[4] As a result, after the relationship termination the expert's only choice is to self-use her remaining information. The game ends when all the information has been utilized.

Some simplifications regarding the expert's information are adopted in the above setup. First, different units of information are equally valuable, which is reflected in the linear payoff functions. Second, information is not re-utilizable in the sense that any part of information can be exploited only once. Third, both self-use and disclosure of information are observable, so the amount of remaining information in each period is commonly known.[5] Finally, information is not cumulative to the investor, therefore whenever an amount of information is disclosed the investor has to utilize it immediately. While these simplifications make our analysis tractable, none of them is essential to our qualitative findings on the dynamics of information disclosure.

The investor may be *aligned* or *biased*. An aligned investor is non-strategic in a pattern that he always cooperates whenever an amount of information is disclosed to him. Conversely, a biased investor is strategic and may betray the expert. The expert is initially uncertain about the investor's type and she holds a prior belief $\mu_0 \in (0, 1)$ that the investor is aligned. Denote by μ_t as the expert's posterior belief in period t. For notational simplicity, we also refer μ and Y as to

simultaneously would not change this equilibrium property, therefore it has no effect on our qualitative findings.

[4] Alternatively, we may assume that even after a betrayal the expert can continue to disclose information to the investor, but information disclosure after a betrayal exerts a lump-sum cost which is high enough to outweigh any benefit from potential cooperation by the investor. Thus, in equilibrium the expert optimally chooses to self-use her remaining information after a betrayal. In many situations, for instance, it is emotionally unacceptable and thus costly for a person to seek further cooperation from another one who has previously betrayed her.

[5] We may assume that only the outcomes of information utilization are observable. However, because there is one to one mapping between the actions and the outcomes, it is without loss of generality to assume that the actions are observable.

the expert's belief and information when explicit indication of time can be omitted.

A history at the beginning of period t summarizes all the actions that have been taken by the players up to this period. A strategy of the expert specifies the amount of information she self-uses or discloses in period t as a function of each history, and a strategy of the investor specifies the action he takes in period t as a function of each history and the amount of information disclosed by the expert in this period. The solution concept in this study is Perfect Bayesian Equilibrium (PBE). A strategy profile and a belief updating system consist of an equilibrium if each player's strategy maximizes his/her payoff and if the expert's belief updating follows Bayes' rule whenever possible. In particular, if in period t the expert's belief is μ_t and the biased investor betrays with probability p_t after a positive information disclosure, Bayes' rule requires that after observing a success the belief μ_{t+1} in period $t+1$ follows

$$\mu_{t+1} = \frac{\mu_t}{\mu_t + (1-\mu_t)(1-p_t)},$$

whereas after observing a failure the belief μ_{t+1} drops to 0 in period $t+1$.

In the remainder of this section we introduce some useful assumptions and notations. Let

$$q = \tfrac{\beta_I - \alpha_I}{\delta\beta_I} \quad \text{and} \quad \mathbf{q}^k = q^0 + q^1 + ... + q^k$$

for $k \geq 0$. The superscript "k" in $\mathbf{q}^k$ is a number indicator, whereas the superscript "k" in q^k is the power of q. The role of q is that, if the expert discloses information y in period t and discloses information qy in period $t+1$ (based on a success in period t), the investor is indifferent between betraying in these two consecutive periods, which is an important condition in the analysis of equilibrium.[6] By definition, $\mathbf{q}^k$ is a function of k.

Assumption 1: $\frac{1}{1+q-\delta q} = \frac{\delta\beta_I}{\delta\alpha_I + \beta_I - \alpha_I} > \frac{1}{\alpha_E}$.

This assumption holds when α_E is relatively large; that is, the investor's cooperation is sufficiently appealing to the expert. Intuitively, it guarantees the existence of equilibrium in which information disclosure occurs, which will be clear shortly.

Assumption 2: $\frac{1-q}{1-\delta q} = \frac{\delta\beta_I + \alpha_I - \beta_I}{\delta\alpha_I} < \frac{1}{\alpha_E}$.

This assumption holds if δ is not too close to 1 when $q < 1$.[7] Intuitively, it implies that, because time discounting is costly, the expert prefers immediate self-use of information in period $t = 0$ to permanent cooperation with the investor even when the latter is feasible.

[6] If the investor betrays and terminates the relationship in period t, his payoff is $\beta_I y$. If he cooperates in period t and betrays in period $t+1$, his payoff is $\alpha_I y + \delta\beta_I qy$. When $q = (\beta_I - \alpha_I)/\delta\beta_I$, $\beta_I y = \alpha_I y + \delta\beta_I qy$ holds.

[7] Notice that $\delta q = (\beta_I - \alpha_I)/\beta_I < 1$, so $1 - \delta q > 0$ always holds. If $q \geq 1$, Assumption 2 holds for any $\delta < 1$. But if $q < 1$, Assumption 2 holds only if δ is relatively small.

Finally, let $\overline{k} \in \mathbb{N}$ satisfy the inequalities

$$\frac{\alpha_E}{1+(1-\delta)(\mathbf{q}^{\overline{k}}-1)} > 1 \geq \frac{\alpha_E}{1+(1-\delta)(\mathbf{q}^{\overline{k}+1}-1)}.$$

By Assumption 1, we have $\overline{k} \geq 1$. By Assumption 2, $\overline{k}$ is finite. In the next section we show that the periods of dynamic information disclosure is bounded above by $\overline{k}+1$ in any equilibrium. Notice that $\overline{k}$ increases in α_E, which indicates that, from the expert's perspective, longer process of information disclosure is acceptable if the investor's cooperation is more productive. Conversely, $\overline{k}$ decreases in q. The intuitive reasoning is that, when q is larger, to induce the biased investor's cooperation in a particular period the expert has to shift more information to the following periods. Because of the discounting cost, the larger q is, the shorter a process of information disclosure should be. A decrease of δ has a similar effect as an increase of q on $\overline{k}$.

A direct observation is that, if the players' interaction can only occur in period $t = 0$, the expert discloses all her information to the investor if and only if $\mu_0 \geq 1/\alpha_E$, and her equilibrium payoff is $\mu_0 \alpha_E Y_0$ if $\mu_0 \geq 1/\alpha_E$ and Y_0 otherwise. Because of the potential hold-up effect exerted by the biased investor, the expert's willingness on information disclosure is limited. In the next section we explore how the expert can improve her payoff when a dynamic process of information disclosure is feasible.

3. Equilibrium analysis

We study the joint dynamics of the expert's information disclosure and the investor's type revelation in this section. Particularly, we show that a process of gradual information disclosure enables the expert to alleviate the hold-up effect and thereby increase her payoff.

3.1 Preliminary results

We present some preliminary results in this subsection. A definition is introduced first.

Definition 1 *A k-period scheme starting from period t is a scheme satisfying, with a sequence $y^k = (y_1, ..., y_{k-1}, y_k)$ and $1 \leq l \leq k$, (1) $Y_t = \sum_{l=1}^{k} y_l$; (2) $y_{l+1} = q y_l$ if $k > l \geq 1$; (3) information y_l is disclosed in period $t-1+l$ if the relationship has not been terminated; (4) information $\sum_{j=l}^{k} y_j$ is self-used in period $t+l$ if the relationship is terminated in the previous period.*

By the definition, a k-period scheme essentially describes a strategy of the expert in the continuation game starting from period t. Specifically, such a strategy makes the biased investor being indifferent between betraying in two consecutive periods when $k > 1$. We will show that in equilibrium the expert's information disclosure, if it occurs, follows a k-period scheme.

Denote $\mu_0^* = 1$. We have the following result.

Lemma 1 *There exists a unique sequence of values* $(\mu_0^*, \mu_1^*, ..., \mu_k^*, ..., \mu_{\overline{k}+1}^*)$ *satisfying*

(a) $V^k(\mu_k^*; Y) = V^{k+1}(\mu_k^*; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k-1)} > Y$ *if* $1 \leq k \leq \overline{k}$, *and* $V^k(\mu_k^*; Y) = Y$ *if* $k = \overline{k}+1$, *in which* $V^k(\mu; Y)$ *is recursively defined by*

$$V^1(\mu; Y) = \alpha_E \mu Y, \quad \text{and for } 2 \leq k \leq \overline{k}+1,$$

$$V^k(\mu; Y) = \frac{\min\{\mu, \mu_{k-1}^*\}}{\mu_{k-1}^*}\left(\frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta V^{k-1}(\max\{\mu, \mu_{k-1}^*\}; Y - \frac{Y}{\mathbf{q}^{k-1}})\right) + \left(1 - \frac{\min\{\mu, \mu_{k-1}^*\}}{\mu_{k-1}^*}\right)\delta\left(Y - \frac{Y}{\mathbf{q}^{k-1}}\right).$$

(b) $\mu_0^* = 1 > \mu_1^* > ... > \mu_k^* > ... > \mu_{\overline{k}+1}^* > 0$.

The proof is relegated to Appendix A. For $1 \leq k \leq \overline{k}+1$, the cut-off values μ_k^* will define the evolution of the expert's beliefs after a series of successes in equilibrium, and the value functions $V^k(\mu; Y)$ will define the expert's equilibrium payoff with information disclosure.[8]

For a particular $V^k(\mu; Y)$, the superscript "k" indicates that the expert's information disclosure follows a k-period scheme. For an initial belief μ, the term $\min\{\mu, \mu_{k-1}^*\}/\mu_{k-1}^*$ is the probability that the investor cooperates in the first period of this scheme. In the term associated to this probability, $\alpha_E Y/\mathbf{q}^{k-1}$ is the expert's payoff from the investor's cooperation in the current period, and $\delta V^{k-1}(\max\{\mu, \mu_{k-1}^*\}; Y - Y/\mathbf{q}^{k-1})$ is her discounted payoff from the continuation game after observing the cooperation. On the other hand, the term $1 - \min\{\mu, \mu_{k-1}^*\}/\mu_{k-1}^*$ is the probability that the investor betrays in the first period of this scheme, and the term $\delta(Y - Y/\mathbf{q}^{k-1})$ is the expert's discounted payoff from the continuation game after observing the betrayal.

Lemma 2 *Let* $1 \leq k, l \leq \overline{k}+1$ *and* $k \neq l$. *If* $\mu \in (\mu_k^*, \mu_{k-1}^*)$, *then* $V^k(\mu; Y) > V^l(\mu; Y)$.

The proof is seen in Appendix A. This property of the value functions will indicate that, for each belief μ (except the cut-off values μ_k^*), the process of information disclosure and the expert's equilibrium payoff are uniquely determined.

For $\overline{k} = 2$, we introduce Figure 1 to summarize the results presented in the previous lemmas. The red envelope will capture the expert's equilibrium payoff from her private information as a

[8] Notice that the subscript "k" in μ_k^* has no relation to the time period k. Instead, it only refers to one the numbers $1, ..., \overline{k}, \overline{k}+1$.

function of the belief μ.

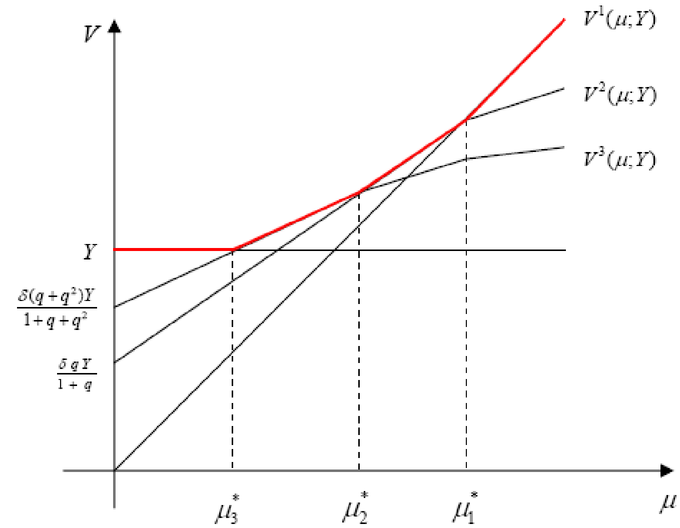

Figure 1: A description of the functions $V^k(\mu;Y)$ and the cut-off values μ_k^*, where $\overline{k}=2$. The red envelope captures the value of dynamic information disclosure as a function of μ.

Consider two beliefs μ and μ', where $\mu \leq \mu'$. Let function $\varphi(\mu;\mu')$ satisfy

$$\mu' = \frac{\mu}{\mu + (1-\mu)(1-\varphi(\mu;\mu'))}.$$

Thus, $\varphi(\mu;\mu')$ is the biased investor's betrayal probability that makes the expert update her belief from μ to μ' after observing a success.

Definition 2 *Let $\mu \in [\mu_k^*, \mu_{k-1}^*]$ and $1 \leq k \leq \overline{k}+1$. A belief path $\boldsymbol{\mu}^k(\mu)$ is a sequence of updated beliefs satisfying $\boldsymbol{\mu}^k(\mu) = (\mu, \mu_{k-1}^*, ..., \mu_1^*, \mu_0^*)$ based on a series of successes.*

Whenever a failure is observed, the expert's belief drops to zero and stays at it forever. In consequence, the only relevant belief updating path is the evolution of beliefs based on a series of successes. We will show that, if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ and $1 \leq k \leq \overline{k}+1$, a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu_0)$ describes the players' behaviors and the expert's belief updating on the equilibrium path of play. Moreover, if the relationship is terminated in period t, the expert should self-use all her remaining information in period $t+1$ to avoid the discounting cost in any equilibrium. From now on we omit the description of strategies and beliefs after observing a failure.

3.2 Equilibrium results

In this subsection we characterize the existence and uniqueness of equilibrium and explain their implications on the players' interactions and payoffs.

Proposition 1 *For any* $\mu_0 \in (0,1)$ *there exists an equilibrium in which the expert's payoff is* $V^k(\mu_0; Y_0)$ *if* $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ *and is* Y_0 *if* $\mu_0 \le \mu_{\overline{k}+1}^*$, *where* $1 \le k \le \overline{k}+1$.

We construct the equilibrium here and relegate the proof to Appendix A. Suppose now the game is in period t with belief $\mu_t = \mu$ and information $Y_t = Y$, and the relationship has not been terminated.

Case 1: $\mu \in [\mu_k^*, \mu_{k-1}^*]$ with $1 \le k \le \overline{k}+1$.[9]

On the equilibrium path. Starting from period t, the expert's strategy follows a k-period scheme, the biased investor's strategy and the expert's belief updating are described by a belief path $\boldsymbol{\mu}^k(\mu)$. Specifically, in period t the expert discloses $Y/\mathbf{q}^{k-1}$ and the biased investor betrays with probability $\varphi(\mu; \mu_{k-1}^*)$. The expert's payoff is $V^k(\mu; Y)$ discounted to period t.

Off the equilibrium path. First, if the biased investor deviates to a probability p' in period t, where $p' \neq \varphi(\mu; \mu_{k-1}^*)$, the expert neglects this deviation and continues to update her belief to μ_{k-1}^* after observing a success.

Second, consider the expert's deviations in period t. There are three cases to consider: (1.1) an amount $y > Y/\mathbf{q}^{k-1}$ is disclosed; (1.2) an amount $y < Y/\mathbf{q}^{k-1}$ is disclosed; (1.3) an amount $x \le Y$ is self-used.[10]

Consider case (1.1). If the expert discloses $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \le l \le k-1$, the biased investor betrays with probability $\varphi(\mu; \mu_{l-1}^*)$. If a success is observed in the current period, (a) if $l = 1$, in the next period the expert discloses all $Y - y$, and (b) if $l > 1$, starting from the next period with probability λ_l the play follows a pair of an $l-1$-period scheme and a belief path $\boldsymbol{\mu}^{l-1}(\mu_{l-1}^*)$, and with probability $1-\lambda_l$ the play follows an l-period scheme and a belief path $\boldsymbol{\mu}^l(\mu_{l-1}^*)$, where λ_l satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_l \beta_I \frac{Y-y}{\mathbf{q}^{l-2}} + (1-\lambda_l)\beta_I \frac{Y-y}{\mathbf{q}^{l-1}}].^{11}$$

Consider case (1.2). If the expert discloses $y \le Y/\mathbf{q}^k$, the biased investor cooperates for sure. Starting from the next period the play follows a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$. If the expert discloses $y \in (Y/\mathbf{q}^k, Y/\mathbf{q}^{k-1})$, the biased investor betrays with probability $\varphi(\mu; \mu_{k-1}^*)$. If a success is observed in the current period, (a) if $k = 1$, in the next period the expert discloses

[9] Notice that for $1 \le k \le \overline{k}$, μ_k^* can be drawn either from $[\mu_k^*, \mu_{k-1}^*]$ or from $[\mu_{k+1}^*, \mu_k^*]$, which implies that if $\mu = \mu_k^*$, we are constructing multiple equilibria for this belief. Similar construction applies to $\mu = \mu_{\overline{k}+1}^*$. The multiplicity of equilibrium at the cut-off values is crucial to pin down the biased investor's responses after the expert's deviations, which would be seen clearly in the construction of the equilibrium and in the proof.

[10] Implicitly, we have $k > 1$ in case (1.1).

[11] It can be verified that $\lambda_l \in [0,1]$ and it increases in y for $y \in (Y_t/\mathbf{q}^l, Y_t/\mathbf{q}^{l-1}]$. The mixing between two schemes that the expert employs here is to keep the biased investor indifferent between betraying and cooperating in period t.

all $Y-y$, and (b) if $k>1$, starting from the next period with probability λ_k the play follows a pair of a $k-1$-period scheme and a belief path $\boldsymbol{\mu}^{k-1}(\mu^*_{k-1})$, and with probability $1-\lambda_k$ the play follows a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu^*_{k-1})$, where λ_k satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_k \beta_I \frac{Y-y}{\mathbf{q}^{k-2}} + (1-\lambda_k)\beta_I \frac{Y-y}{\mathbf{q}^{k-1}}].$$

Consider case (1.3). The investor is inactive in this case and the expert's belief satisfies $\mu_{t+1}=\mu$. Starting from the next period the play follows a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$.

Case 2: $\mu \leq \mu^*_{\overline{k}+1}$.

On the equilibrium path. The expert self-uses all Y in period t and her payoff is Y discounted to this period.

Off the equilibrium path. Only the expert's deviations in period t need to be considered. There are two cases: (2.1) an amount $y \leq Y$ is disclosed; (2.2) an amount $x<Y$ is self-used.

Consider case (2.1). If the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$, the biased investor betrays with probability $\varphi(\mu;\mu^*_{\overline{k}+1})$. If a success is observed in the current period, starting from the next period with probability $\lambda_{\overline{k}+1}$ the play follows a pair of a $\overline{k}+1$-period scheme and a belief path $\boldsymbol{\mu}^{\overline{k}+1}(\mu^*_{\overline{k}+1})$, and with probability $1-\lambda_{\overline{k}+1}$ the expert self-uses all $Y-y$ in period $t+1$, where $\lambda_{\overline{k}+1}$ satisfies

$$\beta_I y = \alpha_I y + \delta \lambda_{\overline{k}+1} \beta_I \frac{Y-y}{\mathbf{q}^{\overline{k}}}.$$

If the expert discloses $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \leq l \leq \overline{k}+1$, the continuation play is the same as specified in case (1.1).

Consider case (2.2). The investor is inactive in this case and the expert's belief satisfies $\mu_{t+1}=\mu$. In the next period the expert self-uses all $Y-x$.

The construction of the equilibrium is ended. To have some intuitive understandings, we illustrate the main features of the equilibrium with a simplified example.

Example 1 *Consider a game with the parameters that* $\overline{k} \geq 2$, $q=1$, $\mu_0 \in (\mu^*_3, \mu^*_2)$ *and* $Y_0 = Y$.

With these parameters, on the equilibrium path the expert discloses information $Y/3$ to the investor in period $t=0$, and continues to disclose $Y/3$ in period $t=1$ and $Y/3$ in period $t=2$ based on a series of successes. In period $t=0$, the biased investor betrays with probability $\varphi(\mu_0;\mu^*_2)$. In period $t=1$, if information is disclosed, he betrays with probability $\varphi(\mu^*_2;\mu^*_1)$. In period $t=2$, if information is disclosed, he betrays for sure.

The key issue in constructing the equilibrium is how the biased investor should response if the expert deviates. Because of the observability of the expert's information disclosure, the concept of PBE requires that, after a deviation of the expert, the players' continuation play and the expert's belief updating should also consist of an equilibrium. Keeping two inquiries in mind is helpful to the understanding of this requirement. The first one is, given that the biased investor is indifferent between cooperating and betraying in period $t = 0$ with disclosed information $Y/3$, should he strictly prefer to betray if the disclosed information is $Y/3 + \epsilon$ in this period, where $\epsilon > 0$ but is arbitrarily small?[12] The answer is no. If he betrays for sure in this case, after observing a success the expert knows that the investor is certain to be aligned and thereby she would disclose all remaining information $2Y/3 - \epsilon$ in the next period. But then the biased investor should cooperate for sure in period $t = 0$ in order to betray in period $t = 1$, which generates a contradiction.

In the equilibrium we construct, in period $t = 0$ if the expert deviates to an amount $y \leq Y/4$, the biased investor cooperates for sure; if the deviation amount is $y \in (Y/4, Y/3)$, the betrayal probability is $\varphi(\mu_0; \mu_2^*)$; if the deviation amount is $y \in (Y/3, Y/2]$, the betrayal probability is $\varphi(\mu_0; \mu_1^*)$; finally, if the deviation amount is $y > Y/2$, the betrayal probability is 1. Figure 2 summarizes the belief updating system for this example.

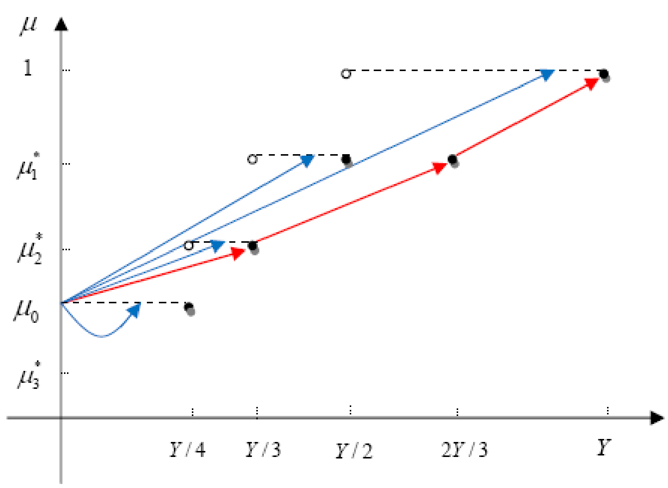

Figure 2: An example of the belief updating system, where $\mu_0 \in [\mu_3^*, \mu_2^*]$, $q = 1$ and $\bar{k} \geq 2$. The red arrows represent the belief updating after cooperation on the equilibrium path. The blue arrows represent the belief updating after cooperation off the equilibrium path. The arrows for the belief updating after betrayal are omitted.

Consider two amounts y and y' of information disclosure by the expert in period $t = 0$, which satisfy $y, y' \in (Y/4, Y/3]$ and $y > y'$. Notice that the biased investor's payoff from a betrayal is strictly higher with y. The second inquiry is, what makes the biased investor use the same betrayal probability $\varphi(\mu_0; \mu_2^*) \in (0, 1)$ in response to the different amounts y and y'? The answer

[12] From now on the term ϵ always refers to "$\epsilon > 0$ but is arbitrarily small".

relies on the multiple equilibria at the cut-off value μ_2^*. In the continuation game that belief reaches μ_2^*, there is an equilibrium in which the expert employs a 2-period scheme and there is another equilibrium in which the expert employs a 3-period scheme. By mixing between these two equilibria, the biased investor can be induced to betray with probability $\varphi(\mu_0; \mu_2^*)$ for any $y, y' \in (Y/4, Y/3]$. The detailed verification is seen in the proof.

Given the biased investor's responses, the expert's task is to choose the process of information disclosure that optimally balances the timing cost and the hold-up effect she faces. In equilibrium, a 3-period scheme maximizes her payoff from information disclosure, which is given by $V^3(\mu_0; Y)$.

The equilibrium we construct presents some of the main findings in this study. First, the expert can mitigate the hold-up problem exerted by the biased investor by disclosing her information gradually. To see this, notice that if information utilization is restricted in an one-shot game, the expert's payoff is $V^1(\mu_0; Y_0)$ if $\mu_0 \geq 1/\alpha_E$ and Y_0 otherwise. In contrast, in the dynamic game if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*] \cap (\mu_{\overline{k}+1}^*, \mu_1^*)$ with $2 \leq k \leq \overline{k}+1$, the expert's equilibrium payoff is $V^k(\mu_0; Y_0)$, which is strictly larger than both $V^1(\mu_0; Y_0)$ and Y_0 by Lemma 1 and 2. Thus, for a non-empty set of initial beliefs the expert can strictly benefit from the dynamics of information disclosure.

Second, if information disclosure occurs, the more aligned the investor, the faster the process of disclosure. This could be seen from that, if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*]$ and $1 \leq k \leq \overline{k}+1$, information disclosure follows a k-period scheme in equilibrium. Intuitively, if the expert intends to induce the investor's cooperation by information disclosure, she trades off between the discounting cost and the rents captured by the biased investor. For $1 \leq k \leq \overline{k}$, this trade-off is resolved by the cut-off values μ_k^* with the indifference conditions

$$V^{k+1}(\mu_k^*; Y) = \frac{\alpha_E Y}{\mathbf{q}^k} + \delta \frac{(\mathbf{q}^k - 1)}{\mathbf{q}^k} V^k(\mu_k^*; Y) = V^k(\mu_k^*; Y).$$

That is, at the cut-off value μ_k^*, given the investor's equilibrium strategies, the expert is indifferent between a k-period scheme, which is less time costly but gives the biased investor greater rents (a payoff of $\beta_I Y/\mathbf{q}^{k-1}$), and a $k+1$-period scheme, which is more time costly but gives the biased investor less rents (a payoff of $\beta_I Y/\mathbf{q}^k$). Specifically, in the $k+1$-period scheme, the biased investor is induced to cooperate with certainty in the first period, and the continuation play after this period follows a k-period scheme with less remained information. In equilibrium, it is optimal for the expert to use a longer process of information disclosure when the investor is less aligned.

Finally, information disclosure occurs if and only if the investor is somewhat aligned; that is, when $\mu_0 \geq \mu_{\overline{k}+1}^*$. If $\mu_0 < \mu_{\overline{k}+1}^*$, compared with the outside option of self-use, even the most effective process of information disclosure is too costly to the expert. In equilibrium, the cut-off value $\mu^*_{\bar{k}+1}$ determined by the indifference condition

$$V^{\bar{k}+1}(\mu^*_{\bar{k}+1}; Y) = Y$$

solves the expert's problem of when information should be disclosed to the investor.

Our second main result is about the equilibrium uniqueness of this game.

Proposition 2 *The equilibrium of this dynamic game is essentially unique.*

The proof is seen in Appendix A. If the initial belief μ_0 satisfies $\mu_0 \in (\mu^*_k, \mu^*_{k-1})$ and $1 \leq k \leq \bar{k}+1$, the expert's information utilization follows a unique pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu_0)$. If $\mu_0 < \mu^*_{\bar{k}+1}$, the expert's information utilization is uniquely determined by self-use. However, the uniqueness of equilibrium is only in the sense of "essential" because multiplicity of equilibrium arises if the initial belief is at the cut-off values and arises in some off equilibrium path of play. Nevertheless, the expert's equilibrium payoff is unique for any initial belief.

The endogenous completion of the expert's information disclosure is the main determinant of the equilibrium uniqueness. In a particular period, if the biased investor betrays with a probability larger than the one he plays on the equilibrium path, in the event of a success the expert becomes more optimistic about the investor's type and would speed up her information disclosure. However, expecting that the process of disclosure is faster in the continuation game after a success, the biased investor should strictly prefer to cooperate in the current period. A similar argument also shows that the biased investor can not betray with a probability less than the one played on the equilibrium path. As a result, given the biased investor's unique response, the expert's problem regarding her information utilization degenerates to a restricted decision problem, which results in a unique process of information disclosure.

4. Extensions

We consider some extensions in this section. Our main focus is how the expert can increase her payoff if she has some or full commitment power in determining her information utilization.

4.1 Committing to a deadline

In many circumstances the time period for information disclosure is limited. For instance, investment opportunities in the stock market may only be valuable before the implementation of some new regulation policies. In this subsection we explore the effects of a deadline on the expert's information utilization and especially, how the expert can benefit from committing to a deadline.

Definition 3 *Let period T be the deadline period, therefore information disclosure is feasible only in period $t \leq T$.*

In other words, at most $T+1$ periods are available for information disclosure.[13] If the deadline period T is reached, the continuation game becomes a one-shot game and the expert discloses all remaining information if and only if her belief satisfies $\mu \geq 1/\alpha_E$. On the other hand, in the previous section we have seen that, if no deadline exists, the expert discloses all remaining information in a single period if and only if her belief satisfies $\mu \geq \mu_1^*$, where $\mu_1^* > 1/\alpha_E$. This difference is central to the understanding of a deadline's effects on the expert's information utilization.

Intuitively, if period T is reached, the expert can not further benefit from gradual information disclosure and thereby her *ex post* payoff is reduced by this restriction. To see this, notice that for a belief in the range $(\mu_{\bar{k}+1}^*, \mu_1^*)$, the expert's payoff from multi-period's information disclosure is strictly larger than her payoff from single-period's disclosure or self-use. However, expecting that in the deadline period the expert is willing to disclose all remaining information even her belief is not sufficiently large, the biased investor can lower his betrayal probabilities in the periods before the deadline, therefore the expert's *ex ante* payoff can be increased. For example, if $T = 1$ and $\mu_0 \in [\max\{\mu_2^*, 1/\alpha_E\}, \mu_1^*)$, by disclosing an amount $Y_0/(1+q) - \epsilon$ and thereby inducing the investor's full cooperation in period $t = 0$, the expert can guarantee a total payoff sufficiently close to $\alpha_E Y_0[1 + \delta q \mu_0]/(1+q)$, which is strictly larger than the equilibrium payoff $V^2(\mu_0; Y_0)$ without a deadline. Our next result generalizes these intuitions.

Proposition 3 *For any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \bar{k}+1$, if the expert commits to a deadline $T = k-1$, there exists an equilibrium in which her payoff is strictly larger than $V^k(\mu_0; Y_0)$.*

How does the presence of a deadline have effects on the expert's information utilization depends on the expert's initial belief about the investor's type. In the equilibrium we construct in the proof, for any deadline $T = k-1$ and $2 \leq k \leq \bar{k}+1$, the belief set $(0,1)$ is partitioned into three intervals by two cut-off values $\underline{\mu}_k$ and $\bar{\mu}_k$, which satisfy $0 < \underline{\mu}_k < \bar{\mu}_k < 1$. If $\mu_0 \geq \bar{\mu}_k$, the expert's information disclosure follows an l-period scheme, where $1 \leq l < k$, which never reaches the deadline. The reason is, although such a scheme requires that the expert's belief is at least μ_1^* for her to disclose all remaining information in the lth period of this scheme, slowing down the process of disclosure until the deadline is reached is too time costly. For example, if the initial belief satisfies $\mu_0 > \mu_1^*$, the expert discloses all her information in period $t = 0$ no matter what the deadline is. If $\mu_0 \in [\underline{\mu}_k, \bar{\mu}_k]$, the expert's information disclosure follows a k-period scheme. The

[13] The expert's self-use of information is not limited by the deadline T. Thus, even in period $t > T$, self-use of information is feasible.

reason is, for this range of initial beliefs, a scheme reaching the deadline (contingent on a series of successes) can effectively lower the biased investor's betrayal probabilities during the process of disclosure. Finally, if $\mu_0 \leq \underline{\mu}_k$, the expert self-uses her information. Intuitively, if the initial belief is relatively low and for the expert to update her belief to at least $1/\alpha_E$ in the limited time (at most k periods), the betrayal probabilities during the process of disclosure need to be sufficiently large, which makes information disclosure unattractive. For example, if $k = 2$ but $\overline{k}$ is relatively large, $\mu^*_{\overline{k}+1} < \underline{\mu}_2$ holds and thereby the expert with a belief $\mu_0 \in [\mu^*_{\overline{k}+1}, \underline{\mu}_2)$ is crowded out of information disclosure by the presence of a deadline.

Most importantly, if the expert has the ability to commit to a deadline, her payoff can be strictly improved if the initial belief satisfies $\mu_0 \in [\mu^*_k, \mu^*_{k-1})$ and $2 \leq k \leq \overline{k} + 1$. The reasoning is straightforward. Compared with the k-period scheme in the equilibrium without a deadline, now by choosing a deadline $T = k - 1$, the same k-period scheme of information disclosure can induce the biased investor to be more cooperative and therefore increase the expert's payoff.

We describe the expert's belief updating system with a deadline $T = 2$ in the following figure. If $\mu_0 \geq \overline{\mu}_3$, the expert employs a 1-period scheme or a 2-period scheme of information disclosure. If $\mu_0 \in [\underline{\mu}_3, \overline{\mu}_3]$, the expert employs a 3-period scheme that reaches the deadline based on a series of successes. Finally, if $\mu_0 \leq \underline{\mu}_3$, the expert self-uses her information. Notice that in this example $[\mu^*_3, \mu^*_2] \in [\underline{\mu}_3, \overline{\mu}_3]$.

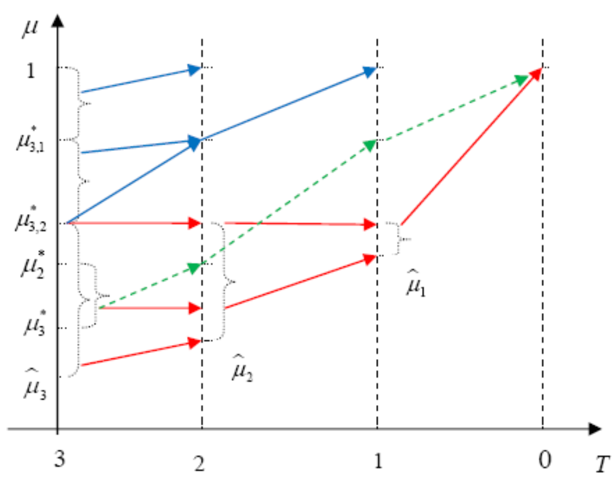

Figure 3: An example of the belief updating system with $T = 3$. The red arrows represent the belief updating after cooperation such that the deadline is reached. The blue arrows represent the belief updating after cooperation such that the deadline is not reached. The green arrows represent the belief updating after cooperation if no deadline exists. The arrows for the belief updating after betrayal are omitted.

The property of payoff improvement is not limited to the set $[\mu^*_{\overline{k}+1}, \mu^*_1)$ of initial beliefs. For example, in the proof we show that $\underline{\mu}_{\overline{k}+1} < \mu^*_{\overline{k}+1}$. As a result, the expert with an initial belief

$\mu_0 \in (\underline{\mu}_{\overline{k}+1}, \mu^*_{\overline{k}+1})$ can also benefit from gradual information disclosure by committing to a deadline $T = \overline{k}$.

4.2 Optimal commitment

Instead of merely committing to a deadline, in some circumstances the expert can commit to a sequence $y^\tau = (y_0, y_1, ...y_\tau)$ of information disclosure, in which y_t is the amount of information to be disclosed in period $t \leq \tau$ contingent on the relationship is ongoing, and τ may or may not be finite.[14] For instance, an auto company's technology transfer to its partner may follow a pre-committed schedule. We explore the properties of the expert's optimal commitment in this section.

Define

$$\overline{V}^1(\mu; Y) = \alpha_E \mu Y,$$

and for $k \geq 2$,

$$\overline{V}^k(\mu; Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta q \frac{\mathbf{q}^{k-2}}{\mathbf{q}^{k-1}} \overline{V}^{k-1}(\mu; Y).$$

Notice that the superscript "k" also refers to the notion of "a k-period scheme".

We simplify the expert's optimal commitment problem with the following arguments. First, because of the linearity of her payoff function, if the expert can benefit from committing to a sequence y^τ, then in the optimal commitment all information should be committed; that is, $\sum_{j=0}^{\tau} y_j = Y_0$. Second, given the optimally committed sequence y^τ, the biased investor should cooperate for sure in any period $t < \tau$. The reason is, if the biased investor betrays with probability $p_t = 1$ and terminates the relationship in period $t < \tau$, the designing of the sub-sequence $(y_{t+1}, y_{t+2}, ...y_\tau)$ has no chance to induce his cooperation and therefore the expert is better off if she re-allocates the amount $\sum_{j=t+1}^{\tau} y_j$ proportionally to the amounts in the sub-sequence $(y_0, y_1, ...y_t)$. On the other hand, if the biased investor betrays with probability $p_t \in (0, 1)$ in period $t < \tau$, it is better for the expert to disclose only $y_t - \epsilon$ in period t for the sake of the investor's full cooperation, and re-allocates ϵ proportionally to the future periods. It can be verified that, in the limit, the expert's optimal commitment is reduced to the following problem:

$$\max_{k \in \mathbb{N}} \overline{V}^k(\mu_0; Y_0)$$

$$s.t. \overline{V}^k(\mu_0; Y_0) \geq Y_0.$$

That is, the expert optimally commits to a k-period scheme of information disclosure, contingent

[14] Notice that in this dynamic game, committing to a sequence y^τ is equivalent to say that the principal has full commitment.

on that her payoff from this scheme is larger than the payoff from self-use. Let $k^*(\mu_0)$ denote the solution to this problem. We have the next result.

Lemma 3 *For any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \le k \le \overline{k}+1$, with the optimal commitment, the expert's payoff is strictly larger than $V^k(\mu_0; Y_0)$.*

The key property of the optimal commitment is that, upon the final period of the $k^*(\mu_0)$-period scheme is reached, the expert agrees to disclose all remaining information no matter what her belief about the investor's type is in this period. Such a commitment enables the expert to induce the biased investor's full cooperation during the first $k^*(\mu_0) - 1$ periods of information disclosure, which is qualitatively similar to, but effectively stronger than the scenario of committing to a deadline.

Given this property, in determining the optimal commitment the expert essentially trades off between the length of the disclosure process and the amount of information to be captured by the biased investor in the final period. In the proof we show that, if the expert's initial belief is moderate, say $\mu_0 \in [\mu_{\overline{k}+1}^*, \mu_1^*)$, she can strictly benefit from her optimal commitment. Moreover, $k^*(\mu_0)$ (weakly) decreases in μ_0, which indicates that the higher the betrayal probability is in the final period, the longer the process of information disclosure should be. Besides, for any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \le k \le \overline{k}+1$, $k^*(\mu_0) \le k$ and the inequality is strict for some initial beliefs μ_0. In consequence, the process is (weakly) faster in the optimal commitment than in the scenario without commitment. Finally, we show that the expert may benefit from information disclosure even her initial belief μ_0 is less than $\mu_{\overline{k}+1}^*$.

5. Conclusions

In this paper, we study a dynamic game in which an expert can utilize her private information either by self-use or by information disclosure to an investor. While the investor has the potential to realize the value of the expert's information in a more efficient way, he has incentive to hold up the expert for his own benefit. Our main finding is that, in the unique equilibrium the expert can mitigate the investor's hold-up effect by a process of gradual information disclosure. We also address how the expert can increase her equilibrium payoff by committing to a deadline or by committing to a particular pattern of information disclosure.

We briefly discuss some assumptions adopted in this study. First, if the expert has no outside option or Assumption 2 does not hold, then in equilibrium the process of information disclosure goes to infinity when the expert's initial belief converges to zero.[15] However, such a process is less realistic because of the requirement that the amounts of information disclosed in the very beginning (or in the very end) should also converge to zero if $q \geq 1$ (or $q < 1$). Second, if the expert's information is not equally valuable or perfectly divisible, then the ordering of different pieces of information to be disclosed may also matter in equilibrium. Finally, given the aligned investor's non-strategic behavior, assuming that information is not cumulative to the investor is without loss of generality. The reason is, any action on information accumulation would reveal the investor's type as biased and thereby cause the stopping of the expert's information disclosure.[16] Because of time discounting and linear payoff function, the biased investor can not benefit from information accumulation.

One potential extension of this game is to consider the case that the investor's action is not observable and his cooperation can only generate a success with probability less than one. In other words, the expert's information utilization is not only subject to adverse selection but also to moral hazard. As a result, after obtaining a payoff zero in a period the expert's belief does not drop to zero. Our conjecture is that there exists a positive value of belief such that, if the expert's belief is less than this value, she resorts to self-use of information. However, a complete characterization of this setup is more complicated.

Another extension is to consider the case that the amount Y_0 of information is initially unknown to the investor. For instance, the expert may be of a low type $Y_0 = Y_L > 0$ with probability $\theta \in (0, 1)$, and may be of a high type $Y_0 = Y_H > Y_L$ with probability $1 - \theta$. The dynamics of information disclosure becomes more subtle because of the type pooling and separating of the experts. Intuitively, a low type of expert may have attempt to pretend to be a high type in order to delay the investor's betrayal, whereas a high type of expert may mimic a low type's behavior to fasten the investor's type revelation. Alternatively, we may also assume that the amount of information self-used by the expert is not observable to the investor. In this case, the expert may use her outside option more strategically because of the endogenous generation of private types. For instance, it is not necessarily true that whenever the expert prefers to self-use *some* information, she self-uses *all* information immediately. We leave these questions open for future research.

[15] Technically, the construction of cut-off values μ_k^* and value functions $V^k(\mu; Y)$ in Lemma 1 is not restricted by the condition $V^k(\mu; Y) \geq Y$ and we have an infinite sequence such that $k \to +\infty$ and $\mu_k^* \to 0$.

[16] Given Assumption 2, it can be verified that if $\mu = 0$, the expert should self-use her information instead of seeking cooperation from the biased investor.

Appendix A. Proofs.

The proof of Lemma 1.

Proof. The proof is by induction. Consider $V^1(\mu;Y)$ and $V^2(\mu;Y)$ for any possible value $\mu_1^* \in (0,1]$. We have $V^1(0;Y) < V^2(0;Y)$ and $V^1(1;Y) > V^2(1;Y)$. Notice that both of the value functions strictly increase in μ given the condition $\overline{k} \geq 1$, but the slope of $V^1(\mu;Y)$ is strictly larger than the slope of $V^2(\mu;Y)$ for any μ,[17] thus we have a unique $\mu_1^* = \frac{1}{1+q-\delta q}$ satisfying $V^1(\mu_1^*;Y) = V^2(\mu_1^*;Y) = \frac{\alpha_E Y}{1+q-\delta q} > Y$. Moreover, $V^1(\mu;Y) > V^2(\mu;Y)$ for $\mu > \mu_1^*$, and $V^1(\mu;Y) < V^2(\mu;Y)$ for $\mu < \mu_1^*$.

Now suppose that for any $k-1 = 1,\cdots,\overline{k}-1$ there is a unique μ_{k-1}^* satisfying (a) and (b). We show that there is a unique μ_k^* also satisfying (a) and (b). First we have

$$V^k(0;Y) = \delta\frac{(\mathbf{q}^{k-1}-1)Y}{\mathbf{q}^{k-1}} < \delta\frac{(\mathbf{q}^{k}-1)Y}{\mathbf{q}^{k}} = V^{k+1}(0;Y).$$

Second, consider $\mu = \mu_{k-1}^*$. Suppose a value $\mu_k^* \leq \mu_{k-1}^*$, we have

$$\begin{aligned}
V^{k+1}(\mu_{k-1}^*;Y) - V^k(\mu_{k-1}^*;Y) &= \frac{\alpha_E Y}{\mathbf{q}^k} + \delta V^k(\mu_{k-1}^*;\frac{(\mathbf{q}^k-1)Y}{\mathbf{q}^k}) - V^k(\mu_{k-1}^*;Y) \\
&= \frac{\alpha_E Y}{\mathbf{q}^k} + (\delta\frac{(\mathbf{q}^k-1)}{\mathbf{q}^k} - 1)V^k(\mu_{k-1}^*;Y) \\
&= \frac{\alpha_E Y}{\mathbf{q}^k}(1 - \frac{1+(1-\delta)(\mathbf{q}^k-1)}{1+(1-\delta)(\mathbf{q}^{k-1}-1)}) \\
&< 0,
\end{aligned}$$

in which the second equality applies because $V^k(\mu_{k-1}^*;Y)$ is linear in Y. Because for any possible value $\mu_k^* \in (0,\mu_{k-1}^*]$ both $V^k(\mu;Y)$ and $V^{k+1}(\mu;Y)$ strictly increase in μ and $V^k(\mu;Y)$ has a larger slope, there is a unique $\mu_k^* \in (0,\mu_{k-1}^*)$ satisfying (1) $V^{k+1}(\mu_k^*;Y) = V^k(\mu_k^*;Y)$, (2) $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu \in (\mu_k^*,\mu_{k-1}^*]$, and (3) $V^k(\mu;Y) < V^{k+1}(\mu;Y)$ for $\mu < \mu_k^*$. Moreover, the statement of (2) can be augmented in a way that $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu > \mu_k^*$. To see this, consider $\mu \in [\mu_{k-1}^*,\mu_{k-2}^*]$. We have

$$V^k(\mu;Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta\frac{(\mathbf{q}^{k-1}-1)}{\mathbf{q}^{k-1}}V^{k-1}(\mu;Y)$$

and

$$V^{k+1}(\mu;Y) = \frac{\alpha_E Y}{\mathbf{q}^k} + \frac{\delta\alpha_E qY}{\mathbf{q}^k} + \delta^2\frac{(\mathbf{q}^k-1-q)}{\mathbf{q}^k}V^{k-1}(\mu;Y).$$

[17]Technically, the slope of $V^2(\mu;Y)$ at the kink μ_1^* is not defined. However, because $V^2(\mu;Y)$ is continuous in μ, this kink does not affect the comparison between these two value functions, we omit this special case. Similar treatment is applied to the other value functions.

By the conditions that $\alpha_E > 1 + (1-\delta)(\mathbf{q}^k - 1)$ and $V^{k-1}(\mu;Y) \geq \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^{k-1}-1)}$, it is direct to see that $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu \in [\mu^*_{k-1}, \mu^*_{k-2}]$. By applying this argument recursively, we have that $V^k(\mu;Y) > V^{k+1}(\mu;Y)$ for $\mu > \mu^*_k$.

Moreover, because

$$V^{k+1}(\mu^*_k;Y) = \frac{\alpha_E Y}{\mathbf{q}^k} + \delta\frac{(\mathbf{q}^k - 1)}{\mathbf{q}^k}V^k(\mu^*_k;Y) = V^k(\mu^*_k;Y),$$

we can solve

$$V^k(\mu^*_k;Y) = V^{k+1}(\mu^*_k;Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)} > Y,$$

in which the inequality is because $\alpha_E > 1 + (1-\delta)(\mathbf{q}^k - 1)$ for $k \leq \overline{k}$.

Now consider $V^{\overline{k}+1}(\mu;Y)$. Because

$$V^{\overline{k}+1}(0;Y) = \delta\frac{(\mathbf{q}^{\overline{k}} - 1)Y}{\mathbf{q}^{\overline{k}}} < Y,$$

$$V^{\overline{k}+1}(\mu^*_{\overline{k}};Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^{\overline{k}} - 1)} > Y,$$

and $V^{\overline{k}+1}(\mu;Y)$ strictly increases in μ for any $\mu \in [0, \mu^*_{\overline{k}}]$, there is a unique $\mu^*_{\overline{k}+1} \in (0, \mu^*_{\overline{k}})$ satisfying $V^{\overline{k}+1}(\mu^*_{\overline{k}+1};Y) = Y$. It can be further verified that $V^{\overline{k}+1}(\mu;Y) > Y$ for $\mu > \mu^*_{\overline{k}+1}$.

Moreover, we can solve the cut-off values μ^*_k explicitly. For $1 \leq k \leq \overline{k}$, let

$$\Phi^k = \mathbf{q}^k\alpha_E - \delta(\mathbf{q}^k - 1)(1 + (1-\delta)(\mathbf{q}^{k+1} - 1))$$

and

$$\Psi^k = \mathbf{q}^k\alpha_E - \delta(\mathbf{q}^k - 1)(1 + (1-\delta)(\mathbf{q}^k - 1)),$$

we have $0 < \Phi^k < \Psi^k$ because $\alpha_E > 1+(1-\delta)(\mathbf{q}^k-1)$ for $k \leq \overline{k}$. Notice that μ^*_1 can be represented as $\mu^*_1 = \frac{1}{1+q-\delta q}\frac{\Phi^0}{\Psi^0}$. For $2 \leq k \leq \overline{k}$, By the condition

$$V^k(\mu^*_k;Y) = \frac{\mu^*_k}{\mu^*_{k-1}}(\alpha_E\frac{Y}{\mathbf{q}^{k-1}} + \delta V^{k-1}(\mu^*_{k-1}; Y - \frac{Y}{\mathbf{q}^{k-1}})) + (1 - \frac{\mu^*_k}{\mu^*_{k-1}})\delta(Y - \frac{Y}{\mathbf{q}^{k-1}}) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)},$$

we can solve μ^*_k recursively as

$$\mu^*_k = \frac{1}{1+(1-\delta)(\mathbf{q}^k - 1)}\prod_{j=0}^{k-1}\frac{\Phi^j}{\Psi^j}.$$

Finally, for $k = \overline{k}+1$, by the condition

$$V^{\overline{k}+1}(\mu^*_{\overline{k}+1};Y) = \frac{\mu^*_{\overline{k}+1}}{\mu^*_{\overline{k}}}(\alpha_E\frac{Y}{\mathbf{q}^{\overline{k}}} + \delta V^{\overline{k}}(\mu^*_{\overline{k}}; Y - \frac{Y}{\mathbf{q}^{\overline{k}}})) + (1 - \frac{\mu^*_{\overline{k}+1}}{\mu^*_{\overline{k}}})\delta(Y - \frac{Y}{\mathbf{q}^{\overline{k}}}) = Y,$$

we have

$$\mu^*_{\bar{k}+1} = \frac{1+(1-\delta)(\mathbf{q}^{\bar{k}}-1)}{\Psi^{\bar{k}}} \prod_{j=0}^{\bar{k}-1} \frac{\Phi^j}{\Psi^j}.$$

■

The proof of Lemma 2.

Proof. Consider $k \leq \bar{k}$ first. In the proof of the previous lemma we have seen that, if $l = k+1$, $V^k(\mu^*_k; Y) = V^{k+1}(\mu^*_k; Y)$ and $V^k(\mu; Y) > V^{k+1}(\mu; Y)$ for any $\mu > \mu^*_k$. Now consider $l = k+2$ (in case $k+2 \leq \bar{k}$). We have seen that $V^{k+1}(\mu^*_{k+1}; Y) = V^{k+2}(\mu^*_{k+1}; Y)$ and $V^{k+1}(\mu; Y) > V^{k+2}(\mu; Y)$ for $\mu > \mu^*_{k+1}$. Because $\mu \in (\mu^*_k, \mu^*_{k-1})$ and $\mu^*_k > \mu^*_{k+1}$, by transitivity we have $V^k(\mu; Y) > V^{k+2}(\mu; Y)$. Recursively, we can show that for any $l > k$, the inequality $V^k(\mu; Y) > V^l(\mu; Y)$ holds for any $\mu \in (\mu^*_k, \mu^*_{k-1})$.

By applying a similar argument, we can show that for any $k \leq \bar{k}+1$ and any $l < k$, $V^k(\mu; Y) > V^l(\mu; Y)$ holds for any $\mu \in (\mu^*_k, \mu^*_{k-1})$. ■

The proof of Proposition 1.

We introduce a technical result here, which will be used in the proof of the equilibrium.

Lemma A1 *For $\mu \in [\mu^*_k, \mu^*_{k-1}]$ and information Y, with $2 \leq k \leq \bar{k}+1$ and $2 \leq j \leq k$, we have the following inequality*

$$V^j(\mu; Y) > \frac{\mu}{\mu^*_{j-2}}(\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + \delta V^{j-1}(\mu^*_{j-2}; Y - \frac{Y}{\mathbf{q}^{j-1}})) + (1 - \frac{\mu}{\mu^*_{j-2}})\delta(Y - \frac{Y}{\mathbf{q}^{j-1}}).$$

Proof. Let $Z^j(\mu; Y)$ denote the term on the right side of the inequality. By extending the value function $V^{j-1}(\cdot; \cdot)$, we have

$$\begin{aligned} V^j(\mu; Y) &= \frac{\mu}{\mu^*_{j-1}}\delta[\frac{\mu^*_{j-1}}{\mu^*_{j-2}}(\alpha_E \frac{qY}{\mathbf{q}^{j-1}} + \delta V^{j-2}(\mu^*_{j-2}; Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})) + (1 - \frac{\mu^*_{j-1}}{\mu^*_{j-2}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})] \\ &\quad + \frac{\mu}{\mu^*_{j-1}}\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + (1 - \frac{\mu}{\mu^*_{j-1}})\delta(Y - \frac{Y}{\mathbf{q}^{j-1}}) \end{aligned}$$

and

$$\begin{aligned} Z^j(\mu; Y) &= \frac{\mu}{\mu^*_{j-2}}\delta[\frac{\mu^*_{j-2}}{\mu^*_{j-2}}(\alpha_E \frac{qY}{\mathbf{q}^{j-1}} + \delta V^{j-2}(\mu^*_{j-2}; Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})) + (1 - \frac{\mu^*_{j-2}}{\mu^*_{j-2}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^{j-1}})] \\ &\quad + \frac{\mu}{\mu^*_{j-2}}\alpha_E \frac{Y}{\mathbf{q}^{j-1}} + (1 - \frac{\mu}{\mu^*_{j-2}})\delta(Y - \frac{Y}{\mathbf{q}^{j-1}}). \end{aligned}$$

Thus,

$$V^j(\mu;Y)-Z^j(\mu;Y)=(\frac{\mu}{\mu^*_{j-1}}-\frac{\mu}{\mu^*_{j-2}})[\alpha_E\frac{Y}{\mathbf{q}^{j-1}}+\delta^2(Y-\frac{(1+q)Y}{\mathbf{q}^{j-1}})-\delta(Y-\frac{Y}{\mathbf{q}^{j-1}})]>0$$

because $\mu^*_{j-1}<\mu^*_{j-2}$ and $\alpha_E>1+(1-\delta)(\mathbf{q}^{\overline{k}}-1)$. ■

The proof of the equilibrium.

Proof. Because of the observability of the expert's actions, a crucial requirement in constructing the equilibrium is that, after a deviation of the expert, the players' continuation play and the expert's belief updating should also consist of an equilibrium. Specifically, whether being on or off the equilibrium path, the biased investor's choice to cooperate or betray not only depends on the amount of information disclosed in the current period, but also on the expected payoff he can obtain in the continuation game.

Case 1: $\mu\in[\mu^*_k,\mu^*_{k-1}]$ with $1\le k\le\overline{k}+1$.

On the equilibrium path, the expert's information disclosure follows a k-period scheme. If $k=1$ so the expert discloses all Y in period t, it is optimal for the biased investor to betray with probability $\varphi(\mu;\mu^*_0)=1$. If $k>1$, the biased investor is indifferent between betraying in period t with a payoff $\beta_IY/\mathbf{q}^{k-1}$ and betraying in period $t+1$ with a total payoff $\alpha_IY/\mathbf{q}^{k-1}+\delta\beta_IqY/\mathbf{q}^{k-1}$, so his betrayal probability $\varphi(\mu;\mu^*_{k-1})$ is optimal. Moreover, the expert's belief updating follows Bayes' rule and her payoff is $V^k(\mu;Y)$.

Consider the players' deviations. Notice that off the equilibrium path the expert's belief updating is not required to follow the Bayes' rule, so she can omit any deviation by the biased investor and continue to update her belief as she does on the equilibrium path. Given such a response by the expert, there is no profitable deviation from the probability $\varphi(\mu;\mu^*_{k-1})$ for the biased investor.

Consider case (1.1) that the expert deviates to $y\in(Y/\mathbf{q}^l,Y/\mathbf{q}^{l-1}]$, where $1\le l\le k-1$. If $l=1$, it is direct to verify that the biased investor should betray with probability $\varphi(\mu;\mu^*_0)=1$ and after observing a success the expert should disclose $Y-y$ in period $t+1$. Given these responses, the expert's deviation payoff, which is denoted by D, is

$$D=\mu[\alpha_Iy+\delta\alpha_I(Y-y)]+(1-\mu)\delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y=Y/(1+q)+\epsilon$ or $y=Y$.[18] If $y=Y/(1+q)+\epsilon$, by Lemma A1 and Lemma 2, we have

$$D\le\lim_{\epsilon\downarrow 0}D=Z^2(\mu;Y)\le V^2(\mu;Y)\le V^k(\mu;Y)$$

[18] Without further conditions, it is indeterminate whether D increases in y.

with $1 < k \leq \overline{k}+1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. Thus, it is not a profitable deviation for the expert. If $y = Y$, by Lemma 2 we have

$$D = V^1(\mu; Y) \leq V^k(\mu; Y)$$

with $1 < k \leq \overline{k}+1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert.

If $l > 1$, after observing a success the belief is updated to μ_{l-1}^*. Because at this belief there is a continuation equilibrium with an $l-1$-period scheme and there is another continuation equilibrium with an l-period scheme (notice that these two equilibria give the expert the same payoff $V^{l-1}(\mu_{l-1}^*; Y-y)$), by mixing them with probabilities λ_l and $1-\lambda_l$ respectively, the biased investor is indifferent between betraying in period t and betraying in period $t+1$, so it is optimal for him to betray in period t with probability $\varphi(\mu; \mu_{l-1}^*)$. Given these responses, the expert's deviation payoff is

$$D = \frac{\mu}{\mu_{l-1}^*}(\alpha_E y + \delta V^{l-1}(\mu_{l-1}^*; Y-y)) + (1 - \frac{\mu}{\mu_{l-1}^*})\delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y = Y/\mathbf{q}^l + \epsilon$ or $y = Y/\mathbf{q}^{l-1}$. If $y = Y/\mathbf{q}^l + \epsilon$, by Lemma A1 and Lemma 2 we have

$$D \leq \lim_{\epsilon \downarrow 0} D = Z^{l+1}(\mu; Y) \leq V^{l+1}(\mu; Y) \leq V^k(\mu; Y)$$

with $1 < l \leq k-1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^{l-1}$, by Lemma 2 we have

$$D = V^l(\mu; Y) \leq V^k(\mu; Y)$$

with $1 < l \leq k-1$ and $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert.

Consider case (1.2) that the expert discloses $y < Y/\mathbf{q}^{k-1}$. If $y \leq Y/\mathbf{q}^k$ and the biased investor cooperates for sure in period t, $\mu_{t+1} = \mu_t$ and the play of a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$ consists of an equilibrium starting from period $t+1$. Given this continuation play, the biased investor actually should cooperates for sure because

$$\beta_I y \leq \alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{k-1}$$

for any $y \leq Y_t/\mathbf{q}^k$. Given these responses, if the expert discloses $y \leq Y/\mathbf{q}^k$, her payoff is

$$D = \alpha_E y + \delta V^k(\mu; Y-y).$$

Because D is linear in y, the optimal deviation is $y = 0$ or $y = Y/\mathbf{q}^k$. If $y = 0$, we have

$$D = \delta V^k(\mu; Y) < V^k(\mu; Y),$$

so it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^k$, by Lemma 2 we have

$$D = V^{k+1}(\mu; Y) \leq V^k(\mu; Y)$$

with $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert.

If $y \in [Y/\mathbf{q}^k, Y/\mathbf{q}^{k-1})$, after observing a success the belief is updated to μ_{k-1}^*. Because at this belief there is a continuation equilibrium with a $k-1$-period scheme and there is another continuation equilibrium with a k-period scheme, by mixing them with probabilities λ_k and $1-\lambda_k$ respectively, the biased investor is indifferent between betraying in period t and betraying in period $t+1$, so it is optimal for him to betray in period t with probability $\varphi(\mu; \mu_{k-1}^*)$. Given these responses, the expert's deviation payoff is

$$D = \frac{\mu}{\mu_{k-1}^*}(\alpha_E y + \delta V^{k-1}(\mu_{k-1}^*; Y-y)) + (1 - \frac{\mu}{\mu_{k-1}^*})\delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y = Y/\mathbf{q}^k$ or $y = Y/\mathbf{q}^{k-1} - \epsilon$. If $y = Y/\mathbf{q}^k$, by Lemma 2 we have

$$D \leq V^{k+1}(\mu; Y) \leq V^k(\mu; Y)$$

with $\mu \in [\mu_k^*, \mu_{k-1}^*]$. So it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^{k-1} - \epsilon$, we have

$$D \leq \lim_{\epsilon \downarrow 0} D = V^k(\mu; Y).$$

So it is not a profitable deviation for the expert.

Consider case (1.3) that the expert self-uses an amount $x \leq Y$. Because $\mu_{t+1} = \mu_t$ and the continuation play starting from period $t+1$ is a pair of a k-period scheme and a belief path $\boldsymbol{\mu}^k(\mu)$, in period t the expert's deviation payoff D satisfies

$$D = x + \delta V^k(\mu; Y-x) \leq V^k(\mu; Y),$$

which is because $V^k(\mu; Y) \geq Y$ for $\mu \in [\mu_k^*, \mu_{k-1}^*]$ and $k \leq \overline{k}+1$. So it is not a profitable deviation for the expert.

Case 2: $\mu \leq \mu_{\overline{k}+1}^*$.

On the equilibrium path the investor is inactive and the expert's payoff is Y.

Consider case (2.1) that the expert deviates to disclose $y \leq Y$. If $y \leq Y/\mathbf{q}^{\overline{k}+1}$, after observing a success the belief is updated to $\mu_{\overline{k}+1}^*$. Because at this belief there is a continuation equilibrium with a $\overline{k}+1$-period scheme and there is another continuation equilibrium in which all $Y-y$ is self-used, by mixing them with probabilities $\lambda_{\overline{k}+1}$ and $1-\lambda_{\overline{k}+1}$ respectively, the biased investor is indifferent between betraying and cooperating in period t, so it is optimal for him to betray in period t with probability $\varphi(\mu;\mu^*_{\bar{k}+1})$. Given these responses, the expert's deviation payoff is

$$D = \frac{\mu}{\mu^*_{\bar{k}+1}}\alpha_E y + \delta(Y-y).$$

Because D is linear in y, the optimal deviation is $y=0$ or $y = Y/\mathbf{q}^{\bar{k}+1}$. If $y=0$, we have

$$D = \delta Y < Y,$$

so it is not a profitable deviation for the expert. If $y = Y/\mathbf{q}^{\bar{k}+1}$, we have

$$D - Y = \frac{\mu}{\mu^*_{\bar{k}+1}}\alpha_E \frac{Y}{\mathbf{q}^{\bar{k}+1}} - \frac{1+(1-\delta)(\mathbf{q}^{\bar{k}+1}-1)Y}{\mathbf{q}^{\bar{k}+1}} \leq 0$$

because $\mu \leq \mu^*_{\bar{k}+1}$ and $\alpha_E \leq 1+(1-\delta)(\mathbf{q}^{\bar{k}+1}-1)$. So it is not a profitable deviation for the expert.

If $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$, where $1 \leq l \leq \bar{k}+1$, the proof is similar to the one described in case (1.1), and we can verify that

$$D \leq V^{\bar{k}+1}(\mu;Y) \leq Y$$

for any deviation payoff D. So it is not profitable for the expert.

Consider case (2.2) that the expert self-uses an amount $x < Y$. His deviation payoff satisfies

$$D = x + \delta(Y-x) < Y,$$

so it is not profitable for the expert. ■

The proof of Proposition 2.

Proof. The proof is done by induction. Suppose the game is in period t with belief $\mu_t = \mu$ and information $Y_t = Y$. Denote by $\overline{W}(\mu;Y)$ as the expert's largest equilibrium payoff measured in period t.

Case 1: $\mu \geq \mu^*_{\bar{k}+1}$.

First consider $\mu \geq \mu^*_1$. In any equilibrium, the expert's payoff is no less than $V^1(\mu;Y) = \alpha_E \mu Y$, which is obtained by disclosing all information Y in period t.

Claim 0.1 There is no equilibrium in which the expert's payoff is strictly larger than $V^1(\mu;Y)$.

Suppose not. Then there exists a $\mu' \geq \mu^*_1$ such that there is an equilibrium in which the expert's largest equilibrium payoff satisfies $\overline{W}(\mu';Y) > V^1(\mu';Y)$. First, in this equilibrium the expert does not self-use an amount $x \leq Y$ in period t. Because if she does so, her largest equilibrium payoff should satisfy $\overline{W}(\mu';Y) \leq x + \delta\overline{W}(\mu';Y-x)$, which requires $\overline{W}(\mu';Y) \leq Y < V^1(\mu';Y)$ because of the necessary linearity of $\overline{W}(\mu';Y)$ in Y. Second, in this equilibrium the expert does not disclose an amount $y > Y/(1+q)$ in period t. Because if she does so, the biased investor betrays for sure in this period and the expert's largest equilibrium payoff satisfies

$$\overline{W}(\mu';Y) = \mu'[\alpha_I y + \delta\alpha_I(Y-y)] + (1-\mu')\delta(Y-y) < V^1(\mu';Y),$$

in which the inequality is shown in the proof of the previous proposition.

Thus, if such an equilibrium exists, it is necessary that the expert discloses $y \leq Y/(1+q)$ in period t. Let $S_1 = \{\mu \geq \mu_1^* | \overline{W}(\mu;Y) > V^1(\mu;Y)\}$ and consider $\mu' \in S_1$. Let $\mu'_{t+1} \geq \mu'$ be the belief in period $t+1$ after observing a success in period t, and let $W(\mu'_{t+1}; Y-y)$ be the expert's continuation payoff at belief μ'_{t+1} in this equilibrium, which is measured in period $t+1$. Then the payoff $\overline{W}(\mu';Y)$ satisfies

$$\begin{aligned}\overline{W}(\mu';Y) &= \frac{\mu'}{\mu'_{t+1}}(\alpha_E y + \delta W(\mu'_{t+1}; Y-y)) + (1-\frac{\mu'}{\mu'_{t+1}})\delta(Y-y) \\ &\leq \frac{\mu'}{\mu'_{t+1}}(\alpha_E y + \delta \overline{W}(\mu'_{t+1}; Y-y)) + (1-\frac{\mu'}{\mu'_{t+1}})\delta(Y-y)\end{aligned}$$

for any $y \leq Y/(1+q)$ and $\mu'_{t+1} \geq \mu' \geq \mu_1^*$. Because $\overline{W}(\mu';Y)$ is necessarily linear in y, it reaches the maximum either at $y=0$ or $y = Y/(1+q)$. If $y=0$, we have $\overline{W}(\mu';Y) \leq Y$, which contradicts $\overline{W}(\mu';Y) > V^1(\mu';Y)$. So it is only possible that $y = Y/(1+q)$. Because

$$V^1(\mu';Y) \geq \frac{\mu'}{\mu'_{t+1}}(\alpha_E \frac{Y}{1+q} + \delta V^1(\mu'_{t+1}; \frac{qY}{1+q})) + (1-\frac{\mu'}{\mu'_{t+1}})\delta\frac{qY}{1+q}$$

for any $\mu'_{t+1} \geq \mu' \geq \mu_1^*$, to have $\overline{W}(\mu';Y) > V^1(\mu';Y)$ it is necessary that

$$\overline{W}(\mu'_{t+1}; \frac{qY}{1+q}) \geq W(\mu'_{t+1}; \frac{qY}{1+q}) > V^1(\mu'_{t+1}; \frac{qY}{1+q}),$$

so $\mu'_{t+1} \in S_1$. Recursively, in any future period l with beliefs $\mu'_l \geq \mu'_{l-1}$, where $l > t$, the continuation payoff $W(\mu'_l; Y_l)$ measured in period l with remaining information $Y_l > 0$ should satisfy $W(\mu'_l; Y_l) > V^1(\mu'_l; Y_l)$ and $\mu'_l \in S_1$. However, because in each period the disclosed information is no more than $1/(1+q)$ of the total remaining information, it can be verified that

$$\overline{W}(\mu';Y) \leq \frac{\alpha_E Y}{1+q}\sum_{h=0}^{+\infty}\frac{(\delta q)^h}{(1+q)^h} = V^1(\mu_1^*;Y) \leq V^1(\mu';Y).$$

So S_1 is empty and there is no equilibrium in which the expert's payoff is strictly larger than $V^1(\mu;Y)$ when $\mu \geq \mu_1^*$.

Claim 0.2 If and only if $\mu = \mu_1^*$ that there exists equilibrium in which the expert does not disclose all Y in period t but her payoff is also $V^1(\mu;Y)$.

First, we have shown above that if the expert discloses $y \in (Y/(1+q), Y)$ or self-uses $x \leq Y$ in period t, her payoff is strictly less than $V^1(\mu; Y)$. Second, if there exists such an equilibrium and the expert discloses $y \leq Y/(1+q)$ in period t, her payoff should have the form

$$\overline{W}(\mu'; Y) \leq \frac{\mu}{\mu_{t+1}}(\alpha_E y + \delta V^1(\mu_{t+1}; Y - y)) + (1 - \frac{\mu}{\mu_{t+1}})\delta(Y - y) \leq V^1(\mu'; Y)$$

for any $\mu \geq \mu_1^*$ and an updated belief μ_{t+1} in period $t+1$ after observing a success in period t. It can be verified that the equalities hold simultaneously if and only if $\mu_{t+1} = \mu = \mu_1^*$ and $y = Y/(1+q)$. Notice that for $\mu = \mu_1^*$, we have shown in proposition 1 that there is an equilibrium in which the expert discloses $y = Y/(1+q)$ in the first period.

Claim 0.3 If $\mu < \mu_1^*$, the expert does not disclose all Y in period t, thereby the biased investor's equilibrium payoff is strictly less than $\beta_I Y$.

To see this, notice that if $\mu < \mu_1^*$ and the expert discloses $y = Y/(1+q) - \epsilon$ in period t, the biased investor's betrayal probability p satisfies $p \leq \varphi(\mu; \mu_1^*)$. This is because that if $p > \varphi(\mu; \mu_1^*)$ then after observing a success the belief μ_{t+1} in period $t+1$ satisfies $\mu_{t+1} > \mu_1^*$ and the expert will disclose all $Y - y$ at this belief μ_{t+1}. But then the biased investor strictly prefers to cooperate in period t because $\alpha_I y + \delta\beta_I(Y - y) > \beta_I y$, which contradicts $p > \varphi(\mu; \mu_1^*)$. Given $p \leq \varphi(\mu; \mu_1^*)$, by disclosing y in period t and disclosing $Y - y$ in period $t+1$ if a success is observed in period t, the expert can guarantee a payoff D satisfying

$$D \geq \frac{\mu}{\mu_1^*}(\alpha_E y + \delta V^1(\mu_1^*; Y - y)) + (1 - \frac{\mu}{\mu_1^*})\delta(Y - y) > V^1(\mu; Y)$$

when $\epsilon \to 0$. So there is no equilibrium in which the expert discloses all Y in period t if $\mu < \mu_1^*$.

For $\mu \geq \mu_{\overline{k}+1}^*$ and $1 \leq k \leq \overline{k} + 1$, we introduce some properties.

Property 1 ($P1$) *If $\mu \in (\mu_k^*, \mu_{k-1}^*)$, there is a unique equilibrium, in which the expert's payoff is $V^k(\mu; Y)$ and the biased investor's payoff is $\beta_I Y/\mathbf{q}^{k-1}$.*

Property 2 ($P2$) *If $\mu = \mu_k^*$, there are multiple equilibria, in which the expert's payoff is $V^k(\mu; Y)$ and the biased investor's payoff ranges from $\beta_I Y/\mathbf{q}^k$ to $\beta_I Y/q^{k-1}$ if $1 \leq k \leq \overline{k}$ and ranges from 0 to $\beta_I Y/q^{\overline{k}}$ if $k = \overline{k} + 1$.*

Property 3 ($P3$) *If $\mu < \mu_k^*$, the biased investor's equilibrium payoff is strictly less than $\beta_I Y/\mathbf{q}^{k-1}$.*

We have seen that these properties hold for $\mu \geq \mu_1^*$. Especially, $P2$ holds for $\mu = \mu_1^*$ because the mixing between two equilibria, in one of which a 2-period scheme is employed and in the other an 1-period scheme is employed.

Suppose these properties hold for $\mu \in [\mu_k^*, \mu_{k-1}^*)$ and $1 \leq k \leq \overline{k}$. Now consider $\mu \in [\mu_{k+1}^*, \mu_k^*)$.

Let y be the amount of information disclosed in period t and p be the biased investor's betrayal probability in this period.

Claim 1.1 In any equilibrium the expert's payoff is at least $V^{k+1}(\mu; Y)$.

Step 1.1.1 If $y < Y/\mathbf{q}^k$, then $p \leq \varphi(\mu; \mu_k^*)$. Suppose not, then after observing a success the belief μ' in period $t+1$ satisfies $\mu' > \mu_k^*$. By $P1$, if the biased investor cooperates in period t his total payoff is at least $\alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{k-1}$, which is strictly larger than the payoff $\beta_I y$ by betraying. So he should cooperate for sure in period t. A contradiction.

Step 1.1.2 There exists an $\eta_{k+1} > 0$ such that if $y \in (Y/\mathbf{q}^k - \eta_{k+1}, Y/\mathbf{q}^k)$ then $p = \varphi(\mu; \mu_k^*)$. Suppose not, so $p < \varphi(\mu; \mu_k^*)$ for any $y < Y/\mathbf{q}^k$, then after observing a success the belief μ' in period $t+1$ satisfies $\mu' < \mu_k^*$. If $y = Y/\mathbf{q}^k - \epsilon$ and $\epsilon \to 0$, the biased investor's payoff by betraying in period t converges $\beta_I Y/\mathbf{q}^k$, while by $P3$ his payoff by cooperating in period t is strictly less than $\alpha_I Y/\mathbf{q}^k + \delta\beta_I qY/\mathbf{q}^k = \beta_I Y/\mathbf{q}^k$, so he should betray for sure in period t. A contradiction.

Step 1.1.3 The expert's equilibrium payoff is at least $V^{k+1}(\mu; Y)$. To see this, notice that if $y = Y/\mathbf{q}^k - \epsilon$, where $0 < \epsilon < \eta_{k+1}$, the expert's payoff is

$$\frac{\mu}{\mu_k^*}(\alpha_I y + \delta V^k(\mu_k^*; Y-y)) + (1 - \frac{\mu}{\mu_k^*})\delta(Y-y).$$

This payoff strictly increases in y and converges to $V^{k+1}(\mu; Y)$ when $\epsilon \to 0$. So in equilibrium the expert's payoff is at least $V^{k+1}(\mu; Y)$.

We introduce a new property here, which is stronger than $P3$.

Property $3'$ ($P3'$) *If $\mu < \mu_k^*$, the biased investor's equilibrium payoff is no larger than $\beta_I Y/\mathbf{q}^k$.*

With Claim 1 shown above, this property holds for $\mu < \mu_1^*$. This is because that, if $\mu < \mu_1^*$, then the disclosed information y in period t satisfies $y \leq Y/(1+q)$ in any equilibrium. If the biased investor weakly prefers to betray with y then his equilibrium payoff is $\beta_I y \leq \beta_I Y/(1+q)$. If he strictly prefers to cooperate in period t then the continuation game starting from period $t+1$ is same to the one starting from period t, except the remaining information is $Y-y$ (or $Y-x$ if the expert self-uses x in period t). In any case it can be verified that $P3'$ holds.

Claim 1.2 There is no equilibrium in which the expert's payoff is strictly larger than $V^{k+1}(\mu; Y)$.

Step 1.2.1 Similar to the argument shown for the case $\mu \geq \mu_1^*$, if there is an equilibrium in which the expert self-uses an amount $x \leq Y$ or discloses an amount $y > Y/(1+q)$ in period t, the expert's payoff in such an equilibrium can not be strictly larger than $V^{k+1}(\mu; Y)$. Moreover, if and only if $\mu = \mu_{k+1}^*$ and $x = Y$ there is an equilibrium in which the expert's payoff equals to $V^{k+1}(\mu; Y)$.

Step 1.2.2 For $2 \leq l \leq k$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p = \varphi(\mu; \mu_{l-1}^*)$. To see this, notice that, first, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p \geq \varphi(\mu; \mu^*_{l-1})$. Suppose not, then by $P3'$, the biased investor's payoff is no more than $\alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{l-1}$ by cooperating in period t, which is strictly less than $\beta_I y$ by betraying, so he should betray for sure in period t. A contradiction. Second, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p \leq \varphi(\mu; \mu^*_{l-1})$. Suppose not, then by $P1$, the biased investor's payoff is no less than $\alpha_I y + \delta\beta_I(Y-y)/\mathbf{q}^{l-2}$ by cooperating in period t, which is strictly larger than $\beta_I y$ by betraying, so he should cooperate for sure in period t. A contradiction. Thus, we conclude that if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p = \varphi(\mu; \mu^*_{l-1})$. Modify this argument slightly, we can show that if $y = Y/\mathbf{q}^{l-1}$ then $p \in [\varphi(\mu; \mu^*_{l-1}), \varphi(\mu; \mu^*_{l-2})]$.

Step 1.2.3 For $2 \leq l \leq k$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$ then there is no equilibrium in which the expert's payoff is strictly larger than $V^{k+1}(\mu; Y)$. Suppose not, then the expert's largest payoff in such an equilibrium satisfies

$$\overline{W}(\mu; Y) \leq \frac{\mu}{\mu^*_{l-1}}(\alpha_I y + \delta V^{l-1}(\mu^*_{l-1}; Y-y)) + (1 - \frac{\mu}{\mu^*_{l-1}})\delta(Y-y) \leq V^l(\mu; Y) < V^{k+1}(\mu; Y),$$

which contradicts the requirement that $\overline{W}(\mu; Y) > V^{k+1}(\mu; Y)$. A contradiction.

Step 1.2.4 If $y < Y/\mathbf{q}^k$, then there is no equilibrium in which the expert's payoff is strictly larger than $V^{k+1}(\mu; Y)$. In Claim 1.1 we have seen that it is true for $y < Y/\mathbf{q}^k$ and $p = \varphi(\mu; \mu^*_k)$. Now consider $y < Y/\mathbf{q}^k$ and $p < \varphi(\mu; \mu^*_k)$. Define $S_{k+1} = \{\mu \in [\mu^*_{k+1}, \mu^*_k] | \overline{W}(\mu; Y) > V^{k+1}(\mu; Y)\}$. Similar to the argument shown for the case $\mu \geq \mu^*_1$, we can show that the set S_{k+1} is empty. Moreover, we can show that $\overline{W}(\mu; Y) = V^{k+1}(\mu; Y)$ only if $\mu = \mu^*_{k+1}$ and $y = Y/\mathbf{q}^{k+1}$ for $2 \leq k \leq \overline{k}$.

Step 1.2.5 If $y = Y/\mathbf{q}^k$ then $p = \varphi(\mu; \mu^*_k)$. By $P3'$, we can show that if $y = Y/\mathbf{q}^k$ then $p \geq \varphi(\mu; \mu^*_k)$. However, if $p > \varphi(\mu; \mu^*_k)$, the expert's payoff is strictly less than $V^{k+1}(\mu; Y)$. So in equilibrium it is necessary that $p = \varphi(\mu; \mu^*_k)$ when $y = Y/\mathbf{q}^k$.

We also pin down what η_{k+1} is in Claim 1.1 By $P1$ and $P3'$, It can be verified that if $y < Y/\mathbf{q}^{k+1}$ then $p = 0$ and if $y \in (Y/\mathbf{q}^{k+1}, Y/\mathbf{q}^k]$ then $p = \varphi(\mu; \mu^*_k)$. So $\eta_{k+1} = q^{k+1}Y/(\mathbf{q}^k \cdot \mathbf{q}^{k+1})$.

Thus, if $\mu \in (\mu^*_{k+1}, \mu^*_k)$, the unique equilibrium is consisted by a pair of a $k+1$-period scheme and a belief path $\boldsymbol{\mu}^{k+1}(\mu)$. In this equilibrium the expert's payoff is $V^{k+1}(\mu; Y)$ and the biased investor's payoff is $\beta_I Y/\mathbf{q}^k$. If $\mu = \mu^*_{k+1}$, by Step 1.2.1 and Step 1.2.4, there are multiple equilibria, in which the expert's payoff is $V^{k+1}(\mu; Y)$, while the biased investor's equilibrium payoff ranges from $\beta_I Y/\mathbf{q}^{k+1}$ to $\beta_I Y/\mathbf{q}^k$ if $0 \leq k \leq \overline{k} - 1$ and ranges from 0 to $\beta_I Y/\mathbf{q}^{\overline{k}}$ if $k = \overline{k}$. So properties $P1$ and $P2$ are true for $\mu \geq \mu^*_{\overline{k}+1}$. Moreover, property $P3'$ has also been verified for $\mu \geq \mu^*_{\overline{k}+1}$. The only remaining part is to show that for $\mu < \mu^*_{\overline{k}+1}$ the biased investor's equilibrium payoff is no larger than $\beta_I Y/\mathbf{q}^{\overline{k}+1}$, which will be completed by the proof for the next case.

Case 2: $\mu < \mu^*_{\overline{k}+1}$.

By self-using all Y in period t, the expert can guarantee a payoff Y. We show that if the expert does not self-use all Y in period t, there is no equilibrium in which the expert's payoff is no less than Y. To see this, first we can show that, with an argument similar to the one shown for the case $\mu \geq \mu^*_1$, if the expert only self-uses an amount $x < Y$ or discloses an amount $y > Y/(1+q)$ in period t, her payoff is strictly less than Y. The remainder of the proof considers $y \leq Y/(1+q)$.

Claim 2.1 If the expert discloses $y > Y/\mathbf{q}^{\overline{k}+1}$, her payoff is strictly less than Y.

Step 2.1.1 For $2 \leq l \leq \overline{k}+1$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$ then $p = \varphi(\mu; \mu^*_{l-1})$, and if $y = Y/\mathbf{q}^{l-1}$ then $p \in [\varphi(\mu; \mu^*_{l-1}), \varphi(\mu; \mu^*_{l-2})]$. The proof replicates the one shown in Step 1.2.2.

Step 2.1.2 For $2 \leq l \leq \overline{k}+1$, if $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1}]$ then there is no equilibrium in which the expert's payoff is no less than Y. Suppose not, then the expert's largest payoff in such an equilibrium satisfies

$$\begin{aligned}\overline{W}(\mu; Y) &\leq \frac{\mu}{\mu^*_{l-1}}(\alpha_I y + \delta V^{l-1}(\mu^*_{l-1}; Y-y)) + (1 - \frac{\mu}{\mu^*_{l-1}})\delta(Y-y) \\ &\leq V^l(\mu; Y) < V^{\overline{k}+1}(\mu; Y) < V^{\overline{k}+1}(\mu^*_{\overline{k}+1}; Y) = Y\end{aligned}$$

which contradicts the requirement that $\overline{W}(\mu; Y) \geq Y$. A contradiction.

Claim 2.2 If the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$, her payoff is no larger than Y.

Suppose not. Define $S_{\overline{k}+1} = \{\mu < \mu^*_{\overline{k}+1} | \overline{W}(\mu; Y) > Y\}$. With an argument similar to the one shown for the case $\mu \geq \mu^*_1$, we can show that the set $S_{\overline{k}+1}$ is empty.

Claim 2.3 If the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$, her payoff is strictly less than Y.

Suppose not. Then for some $\mu < \mu^*_{\overline{k}+1}$ there is an equilibrium in which the expert's payoff satisfies $\overline{W}(\mu; Y) = Y$ by disclosing an amount $y \leq Y/\mathbf{q}^{\overline{k}+1}$ in period t. Notice that $\overline{W}(\mu; Y)$ takes the form

$$\overline{W}(\mu; Y) \leq \frac{\mu}{\mu'}(\alpha_E y + \delta\overline{W}(\mu'; Y-y)) + (1 - \frac{\mu}{\mu'})\delta(Y-y),$$

in which μ' is the next period's belief after observing a success. For the equality to hold, it is necessary that (a) $\mu = \mu'$, (b) $y = Y/\mathbf{q}^{\overline{k}+1}$ and (c) $\overline{W}(\mu'; Y-y) = Y-y$. If (c) is realized by self-using an amount $x \leq Y-y$ in period $t+1$, we have seen that it should be the case $x = Y-y$. However, in this case the biased investor should betray for sure in period t, so $\mu' = 1 \neq \mu$, which makes the equality infeasible. If (c) is realized by continuing to disclose, in a recursive way it should be the case that (b') the principal discloses $1/\mathbf{q}^{\overline{k}+1}$ of the remaining information in each period and (a') the biased investor cooperates for sure in each period. However, (a') contradicts with (b') because in period t the biased investor's payoff by betraying is $\beta_I Y/\mathbf{q}^{\overline{k}+1}$, which is strictly larger than the permanent cooperation payoff $\frac{\alpha_I Y_t}{\mathbf{q}^{\overline{k}+1}} \sum_{h=0}^{+\infty} \frac{\delta^h(\mathbf{q}^{\overline{k}+1}-1)^h}{(\mathbf{q}^{\overline{k}+1})^h}$.

Thus, the equilibrium is unique when $\mu < \mu^*_{\overline{k}+1}$, in which the expert self-uses all Y in period t. Because in this case the investor's payoff is zero, property $P3'$ also holds for $\mu < \mu^*_{\overline{k}+1}$.

Finally we pin down the biased investor's response if the expert discloses $y \leq Y/\mathbf{q}^{\overline{k}+1}$. First, his betrayal probability p satisfies $p \geq \varphi(\mu; \mu^*_{\overline{k}+1})$. If not, the next period's belief after a success is strictly less than $\mu^*_{\overline{k}+1}$ and the expert would self-use all $Y - y$ for sure. But then the biased investor should betray for sure in period t. A contradiction. Second, if $y < Y/\mathbf{q}^{\overline{k}+1}$, his betrayal probability p satisfies $p \leq \varphi(\mu; \mu^*_{\overline{k}+1})$. If not, by $P1$ his payoff is at least $\alpha_I y + \delta\beta_I(Y - y)/\mathbf{q}^{\overline{k}}$ by cooperating in period t, which is strictly larger than $\beta_I y$ by betraying. So he should cooperate for sure. A contradiction. Third, if $y = Y/\mathbf{q}^{\overline{k}+1}$, by $P1$ and $P3'$, it can be verified that $p \in [\varphi(\mu; \mu^*_{\overline{k}+1}), \varphi(\mu; \mu^*_{\overline{k}})]$. ■

The proof of Lemma 3.

Proof. Consider the expert's problem:

$$\max_{k \in \mathbb{N}} \overline{V}^k(\mu_0; Y_0) \qquad s.t. \qquad \overline{V}^k(\mu_0; Y_0) \geq Y_0.$$

Let $k^*(\mu_0)$ be the solution to this problem.

Notice that, for $k \geq 2$, the value function $\overline{V}^k(\mu; Y)$ also has the expression

$$\overline{V}^k(\mu; Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} \sum_{l=0}^{k-2} (\delta q)^l + \frac{(\delta q)^{k-1}}{\mathbf{q}^{k-1}} \alpha_E \mu Y.$$

For a particular k, if there is a $\widetilde{\mu}$ satisfying $\overline{V}^{k+1}(\widetilde{\mu}; Y) = \overline{V}^k(\widetilde{\mu}; Y)$, we can verify that $\overline{V}^k(\mu; Y) > \overline{V}^{k+1}(\mu; Y)$ for $\mu > \widetilde{\mu}$ and $\overline{V}^k(\mu; Y) < \overline{V}^{k+1}(\mu; Y)$ for $\mu < \widetilde{\mu}$. Moreover, we can explicitly solve that $\overline{V}^{k+1}(\widetilde{\mu}; Y) = \overline{V}^k(\widetilde{\mu}; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)}$.

Let $\widetilde{k}$ satisfy

$$\overline{V}^{\widetilde{k}}(0; Y) < \overline{V}^{\widetilde{k}+1}(0; Y) \quad \text{and} \quad \overline{V}^{\widetilde{k}+1}(0; Y) \geq \overline{V}^{\widetilde{k}+2}(0; Y).$$

It can be verified that $\widetilde{k} \geq 1$ and for any $1 \leq j \leq \widetilde{k}$, we have $\overline{V}^j(0; Y) < \overline{V}^{j+1}(0; Y)$. The implication of $\widetilde{k}$ is that, for any $k \geq \widetilde{k} + 1$, if there is a $\widetilde{\mu}$ satisfying $\overline{V}^{k+1}(\widetilde{\mu}; Y) = \overline{V}^k(\widetilde{\mu}; Y)$, then $\widetilde{\mu} \leq 0$. Thus, for any $\mu_0 \in (0, 1)$, $k^*(\mu_0) \leq \widetilde{k} + 1$ is necessary for the expert's optimal commitment.

Similar to the proof shown for Lemma 1, it can be verified that, if $\widetilde{k} < \overline{k}$, there is a sequence $(\widetilde{\mu}^{\widetilde{k}+1}, ..., \widetilde{\mu}^k, ...\widetilde{\mu}^1, \widetilde{\mu}^0)$ satisfying (1) $\widetilde{\mu}^k < \widetilde{\mu}^{k-1}$ for $1 \leq k \leq \widetilde{k} + 1$, (2) $\widetilde{\mu}^{\widetilde{k}+1} = 0$, $\widetilde{\mu}^1 = \mu^*_1$, $\widetilde{\mu}^0 = 1$, (3) $\overline{V}^{k+1}(\widetilde{\mu}^k; Y) = \overline{V}^k(\widetilde{\mu}^k; Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k - 1)} > Y$ for $1 \leq k \leq \widetilde{k}$ and $\overline{V}^{\widetilde{k}+1}(0; Y) \geq Y$. Thus, for any $\mu_0 \in [\widetilde{\mu}^k, \widetilde{\mu}^{k-1}]$ and $1 \leq k \leq \widetilde{k} + 1$, $k^*(\mu_0) = k$ and the expert's optimal commitment payoff is $\overline{V}^k(\mu_0; Y_0)$. Moreover, for $2 \leq k \leq \widetilde{k} + 1$ we can show that $\widetilde{\mu}^k < \mu^*_k$ by showing

$\overline{V}^k(\mu_k^*;Y) > V^k(\mu_k^*;Y)$ recursively. Thus, the process of information disclosure with an optimal commitment is (weakly) faster than the process of information disclosure without a commitment.

On the other hand, if $\widetilde{k} \geq \overline{k}$, there is a sequence $(\widetilde{\mu}^{\overline{k}+1}, ..., \widetilde{\mu}^k, ...\widetilde{\mu}^1, \widetilde{\mu}^0)$ satisfying (1) $\widetilde{\mu}^k < \widetilde{\mu}^{k-1}$ for $1 \leq k \leq \overline{k}+1$, (2) $\widetilde{\mu}^{\overline{k}+1} \geq 0$, $\widetilde{\mu}^1 = \mu_1^*$, $\widetilde{\mu}^0 = 1$, (3) $\overline{V}^{k+1}(\widetilde{\mu}^k;Y) = \overline{V}^k(\widetilde{\mu}^k;Y) = \frac{\alpha_E Y}{1+(1-\delta)(\mathbf{q}^k-1)} > Y$ for $1 \leq k \leq \overline{k}$ and $\overline{V}^{\overline{k}+1}(\widetilde{\mu}^{\overline{k}+1};Y) = Y$. Thus, for any $\mu_0 \in [\widetilde{\mu}^k, \widetilde{\mu}^{k-1}]$ and $1 \leq k \leq \overline{k}+1$, $k^*(\mu_0) = k$ and the expert's optimal commitment payoff is $\overline{V}^k(\mu_0;Y_0)$. Similarly, we have $\widetilde{\mu}^k < \mu_k^*$ for $2 \leq k \leq \overline{k}+1$.

No matter $\widetilde{k} < \overline{k}$ or $\widetilde{k} \geq \overline{k}$, for any $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k}+1$, it can be shown recursively that $\overline{V}^k(\mu_0;Y_0) > V^k(\mu_0;Y_0)$. Combined with $\overline{V}^{k^*(\mu_0)}(\mu_0;Y_0) \geq \overline{V}^k(\mu_0;Y_0)$, we have $\overline{V}^{k^*(\mu_0)}(\mu_0;Y_0) > V^k(\mu_0;Y_0)$, which proves the statement in the proposition.

The main difference between the case $\widetilde{k} < \overline{k}$ and the case $\widetilde{k} \geq \overline{k}$ is that, in the first case the expert can strictly benefit from an optimal commitment for any initial belief $\mu_0 \in (0, \mu_1^*)$, whereas in the second case the expert can strictly benefit from an optimal commitment if and only if $\mu_0 \in (\widetilde{\mu}^{\overline{k}+1}, \mu_1^*)$, where $\widetilde{\mu}^{\overline{k}+1}$ may be larger than 0. ■

Appendix B. **The proof of Proposition 3.**

The proof is done by the following lemmas.

Lemma B1 *For $1 \leq k \leq \overline{k}+1$, there exists a unique sequence of values $(\underline{\mu}_{\overline{k}+1}, ..., \underline{\mu}_k, ...\underline{\mu}_1)$ satisfying*

(a) $0 < \underline{\mu}_{\overline{k}+1} < ... < \underline{\mu}_k < ... < \underline{\mu}_1 < 1$.

(b) $\Upsilon^k(\underline{\mu}_k;Y) = Y$ *in which* $\Upsilon^k(\underline{\mu}_k;Y)$ *is recursively defined by*

$$\Upsilon^1(\mu;Y) = \alpha_E \mu Y, \quad \text{and for } 2 \leq k \leq \overline{k}+1,$$

$$\Upsilon^k(\mu;Y) = \frac{\min\{\mu, \underline{\mu}_{k-1}\}}{\underline{\mu}_{k-1}}\left(\frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta\Upsilon^{k-1}(\max\{\mu, \underline{\mu}_{k-1}\}; Y - \frac{Y}{\mathbf{q}^{k-1}})\right) + \left(1 - \frac{\min\{\mu, \underline{\mu}_{k-1}\}}{\underline{\mu}_{k-1}}\right)\delta\left(Y - \frac{Y}{\mathbf{q}^{k-1}}\right).$$

Proof. Consider property (b). By the definition of $\Upsilon^k(\mu;Y)$, we have $\underline{\mu}_1 = 1/\alpha_E$. Recursively, for $2 \leq k \leq \overline{k}+1$, we have

$$\underline{\mu}_k = \frac{1}{(\alpha_E)^k}\prod_{j=0}^{k-1}(1 + (1-\delta)(\mathbf{q}^j - 1)).$$

Because $\alpha_E > 1 + (1-\delta)(\mathbf{q}^{\overline{k}} - 1)$, property (a) holds. ■

Remark B1 *In the equilibrium we will construct shortly, the cut-off values $\underline{\mu}_k$ and the value functions $\Upsilon^k(\mu;Y)$ work as follows. If the number of remaining periods feasible for information disclosure is k, where $1 \leq k \leq \overline{k}+1$, the cut-off value $\underline{\mu}_k$ determined by $\Upsilon^k(\underline{\mu}_k;Y) = Y$ is the* *threshold between information disclosure and self-use of information in the current period; that is, information disclosure occurs if and only if the current belief μ satisfies $\mu \geq \underline{\mu}_k$.*

Lemma B2 *For $1 \leq k \leq \overline{k}+1$, the following properties hold:*

(a) $\Upsilon^k(\mu;Y)$ strictly increases in μ.

(b) $\underline{\mu}_k < \mu_k^*$.

(c) For $\mu \in [\mu_k^*, \mu_{k-1}^*)$, $\Upsilon^1(\mu;Y) = V^1(\mu;Y)$ if $k=1$ and $\Upsilon^k(\mu;Y) > V^k(\mu;Y)$ if $k \geq 2$.

Proof. Consider (a). First notice that $\Upsilon^1(\mu;Y)$ strictly increases in μ. Now suppose that, for $1 \leq k-1 \leq \overline{k}$, $\Upsilon^{k-1}(\mu;Y)$ also strictly increases in μ. Consider $\Upsilon^k(\mu;Y)$. We have

$$\Upsilon^k(\mu;Y) = \frac{\mu}{\underline{\mu}_{k-1}}\frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta(Y - \frac{Y}{\mathbf{q}^{k-1}})$$

if $\mu \leq \underline{\mu}_{k-1}$ and

$$\Upsilon^k(\mu;Y) = \frac{\alpha_E Y}{\mathbf{q}^{k-1}} + \delta\Upsilon^{k-1}(\mu;Y - \frac{Y}{\mathbf{q}^{k-1}})$$

if $\mu > \underline{\mu}_{k-1}$. In either case, $\Upsilon^k(\mu;Y)$ strictly increases in μ.

Consider (b) and (c). First notice that $\underline{\mu}_1 < \mu_1^*$ and $\Upsilon^1(\mu;Y) = V^1(\mu;Y)$ hold. Suppose that both (b) and (c) hold for $\mu \in [\mu_k^*, \mu_{k-1}^*)$ and $1 \leq k \leq \overline{k}$. Now consider $\mu \in [\mu_{k+1}^*, \mu_k^*)$. Here we introduce a value function as

$$\Delta^{k+1}(\mu;Y) = \frac{\mu}{\mu_k^*}(\alpha_E\frac{Y}{\mathbf{q}^k} + \delta\Upsilon^k(\mu_k^*;Y - \frac{Y}{\mathbf{q}^k})) + (1 - \frac{\mu}{\mu_k^*})\delta(Y - \frac{Y}{\mathbf{q}^k}).$$

By the assumption of induction, we have $\Delta^{k+1}(\mu;Y) \geq V^{k+1}(\mu;Y)$. If $\mu \leq \underline{\mu}_k$ and $\mu_k^* \leq \underline{\mu}_{k-1}$ over here, by extending $\Upsilon^{k+1}(\mu;Y)$ and $\Delta^{k+1}(\mu;Y)$ as follows:

$$\begin{aligned}\Upsilon^{k+1}(\mu;Y) &= \frac{\mu}{\underline{\mu}_k}\delta[\frac{\underline{\mu}_k}{\underline{\mu}_{k-1}}(\alpha_E\frac{qY}{\mathbf{q}^k} + \delta\Upsilon^{k-1}(\underline{\mu}_{k-1};Y - \frac{(1+q)Y}{\mathbf{q}^k})) + (1 - \frac{\underline{\mu}_k}{\underline{\mu}_{k-1}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^k})] \\ &\quad + \frac{\mu}{\underline{\mu}_k}\alpha_E\frac{Y}{\mathbf{q}^k} + (1 - \frac{\mu}{\underline{\mu}_k})\delta(Y - \frac{Y}{\mathbf{q}^k}),\end{aligned}$$

$$\begin{aligned}\Delta^{k+1}(\mu;Y) &= \frac{\mu}{\mu_k^*}\delta[\frac{\mu_k^*}{\underline{\mu}_{k-1}}(\alpha_E\frac{qY}{\mathbf{q}^k} + \delta\Upsilon^{k-1}(\underline{\mu}_{k-1};Y - \frac{(1+q)Y}{\mathbf{q}^k})) + (1 - \frac{\mu_k^*}{\underline{\mu}_{k-1}})\delta(Y - \frac{(1+q)Y}{\mathbf{q}^k})] \\ &\quad + \frac{\mu}{\mu_k^*}\alpha_E\frac{Y}{\mathbf{q}^k} + (1 - \frac{\mu}{\mu_k^*})\delta(Y - \frac{Y}{\mathbf{q}^k}),\end{aligned}$$

we have

$$\Upsilon^{k+1}(\mu;Y) - \Delta^{k+1}(\mu;Y) = (\frac{\mu}{\underline{\mu}_k} - \frac{\mu}{\mu_k^*})(\alpha_E\frac{Y}{\mathbf{q}^k} - \delta(Y - \frac{Y}{\mathbf{q}^k}) + \delta^2(Y - \frac{(1+q)Y}{\mathbf{q}^k})) > 0$$

because $\underline{\mu}_k < \mu_k^*$ and $\alpha_E > 1 + (1-\delta)(\mathbf{q}^{\overline{k}} - 1)$. If $\mu > \underline{\mu}_k$ or $\mu_k^* > \underline{\mu}_{k-1}$, by modifying the above argument slightly, we also can show that $\Upsilon^{k+1}(\mu; Y) - \Delta^{k+1}(\mu; Y) > 0$. Thus, we have

$$\Upsilon^{k+1}(\mu; Y) > \Delta^{k+1}(\mu; Y) \geq V^{k+1}(\mu; Y) \geq Y$$

for $\mu \in [\mu_{k+1}^*, \mu_k^*)$ and $1 \leq k \leq \overline{k}$. Because $\Upsilon^{k+1}(\mu; Y)$ strictly increases in μ, we have $\underline{\mu}_{k+1} < \mu_{k+1}^*$. This completes the proof of (b) and (c). ■

Remark B2 *In the equilibrium we will construct, given a deadline $T = k - 1$, the expert's equilibrium payoff is no less than $\Upsilon^k(\mu_0; Y_0)$ if $\mu_0 \geq \underline{\mu}_k$. By properties (b) and (c) shown in this lemma, such an equilibrium payoff is strictly larger than $V^k(\mu_0; Y_0)$ if $\mu_0 \in [\mu_k^*, \mu_{k-1}^*)$ and $2 \leq k \leq \overline{k} + 1$.*

Lemma B3 *For any deadline $T = k - 1$ and $2 \leq k \leq \overline{k} + 1$, there is a number $\bar{l}(k)$ as a function of k and a sequence $(\mu_{k,\bar{l}(k)}^*, \mu_{k,\bar{l}(k)-1}^*, ...\mu_{k,1}^*)$ satisfying*

(a) For $3 \leq k \leq \overline{k}+1$ and $2 \leq l \leq \bar{l}(k)$, $\mu_{k,1}^* = \mu_1^*$, $\underline{\mu}_k < \mu_{k,\bar{l}(k)}^*$, $\mu_{k,l}^* < \mu_{k,l-1}^*$ and $\mu_{k,l}^* < \mu_{k-1,l-1}^*$.

(b) $U^{k,l}(\mu; Y) \gtreqqless U^{k,l+1}(\mu; Y)$ if $\mu \gtreqqless \mu_{k,l}^*$ and $1 \leq l < \bar{l}(k)$, and $U^{k,l}(\mu; Y) \gtreqqless \Upsilon^k(\mu; Y)$ if $\mu \gtreqqless \mu_{k,\bar{l}(k)}^*$ and $l = \bar{l}(k)$, in which $U^{k,l}(\mu; Y)$ is recursively defined by

$$U^{k,1}(\mu; Y) = \mu\alpha_E Y \quad \textit{and for } 2 \leq l \leq \bar{l}(k),$$

$$\begin{aligned} U^{k,l}(\mu; Y) \quad = \quad & \frac{\min\{\mu, \mu_{k-1,l-1}^*\}}{\mu_{k-1,l-1}^*}(\alpha_E \frac{Y}{\mathbf{q}^{l-1}} + \delta U^{k-1,l-1}(\max\{\mu, \mu_{k-1,l-1}^*\}; Y - \frac{Y}{\mathbf{q}^{l-1}})) \\ & +(1 - \frac{\min\{\mu, \mu_{k-1,l-1}^*\}}{\mu_{k-1,l-1}^*})\delta(Y - \frac{Y}{\mathbf{q}^{l-1}}). \end{aligned}$$

For a particular value function $U^{k,l}(\mu; Y)$, the superscript "k" indicates the number $T + 1$ of periods feasible for information disclosure, and the superscript "l" indicates that a l-period scheme is employed in the process of information disclosure.

Proof. Notice that $U^{k,1}(\mu; Y) = U^{k,2}(\mu; Y)$ holds if $\mu = \mu_1^*$. Thus, denote $\mu_{k,1}^* = \mu_1^*$. Also, denote $\mu_{k,0}^* = 1$.

If $T = 1$, $U^{2,1}(\mu_{2,1}^*; Y) = \Upsilon^2(\mu_{2,1}^*; Y)$ holds. We define the sequence by a single element sequence $(\mu_{2,1}^*)$. It can be verified that all the other properties in (a) and (b) hold.

Consider $T = 2$. Notice that $U^{3,2}(\mu; Y) = V^2(\mu; Y)$, so $U^{3,2}(\mu; Y)$ strictly increases in μ. We also have $U^{3,2}(0; Y) < \Upsilon^3(0; Y)$ and $U^{3,2}(\mu; Y) > \Upsilon^3(\mu; Y)$ when $\mu \geq \mu_{2,1}^*$. Because $U^{3,2}(\mu; Y)$ always has a larger slope, there is a unique $\mu_{3,2}^*$ satisfying $U^{3,2}(\mu; Y) = \Upsilon^3(\mu; Y)$ and $0 < \mu_{3,2}^* < \mu_{2,1}^*$. Define the sequence as $(\mu_{3,2}^*, \mu_{3,1}^*)$ and it can be verified that all the other properties in (a) and (b) hold. Moreover, because $\Upsilon^3(\mu_2^*;Y) > V^2(\mu_2^*;Y)$, we have $\mu_{3,2}^* > \mu_2^*$.

Suppose that for $2 \le k \le \overline{k}$ a sequence $(\mu^*_{k,\bar{l}(k)}, \mu^*_{k,\bar{l}(k)-1}, ...\mu^*_{k,1})$ satisfying (a) and (b) has been defined. Now we define the sequence for $k+1$.

First, for any $2 \le j \le \bar{l}(k)$, there is a unique $\mu_{k+1,j}$ satisfying $U^{k+1,j}(\mu;Y) = U^{k+1,j+1}(\mu;Y)$ and $0 < \mu_{k+1,j} < \mu^*_{k,j-1}$. To see this, notice that if $\mu \ge \mu^*_{k,j-1}$ we have

$$U^{k+1,j}(\mu;Y) = \alpha_E \frac{Y}{\mathbf{q}^{j-1}} + \delta U^{k,j-1}(\mu;Y - \frac{Y}{\mathbf{q}^{j-1}})$$

and

$$U^{k+1,j+1}(\mu;Y) = \alpha_E \frac{Y}{\mathbf{q}^{j}} + \delta U^{k,j}(\mu;Y - \frac{Y}{\mathbf{q}^{j}}).$$

By the inductive assumption, $U^{k,j-1}(\mu;Y) \ge U^{k,j}(\mu;Y)$ if $\mu \ge \mu^*_{k,j-1}$, so $U^{k+1,j}(\mu;Y) > U^{k+1,j+1}(\mu;Y)$ if $\mu \ge \mu^*_{k,j-1}$. On the other hand, we have $U^{k+1,j}(0;Y) > U^{k+1,j+1}(0;Y)$. Because both of $U^{k+1,j}(\mu;Y)$ and $U^{k+1,j+1}(\mu;Y)$ strictly increase in μ and the former has a larger slope in μ, there is a unique $\mu_{k+1,j}$ satisfying the conditions.

Second, there is a unique μ' satisfying $U^{k+1,\bar{l}(k)}(\mu;Y) = \Upsilon^{k+1}(\mu;Y)$ and $0 < \mu' < \mu^*_{k,\bar{l}(k)-1}$. To see this, notice that if $\mu \ge \mu^*_{k,\bar{l}(k)-1} > \underline{\mu}_k$, we have

$$\Upsilon^{k+1}(\mu;Y) = \alpha_E \frac{Y}{\mathbf{q}^{k}} + \delta \Upsilon^{k}(\mu;Y - \frac{Y}{\mathbf{q}^{k}}).$$

Because $U^{k,\bar{l}(k)-1}(\mu;Y) \ge U^{k,\bar{l}(k)}(\mu;Y) > \Upsilon^k(\mu;Y)$ if $\mu \ge \mu^*_{k,\bar{l}(k)-1}$, we have $U^{k+1,\bar{l}(k)}(\mu;Y) > \Upsilon^{k+1}(\mu;Y)$ if $\mu \ge \mu^*_{k,\bar{l}(k)-1}$. On the other hand, we have $U^{k+1,\bar{l}(k)}(0;Y) < \Upsilon^{k+1}(0;Y)$. Because both $U^{k+1,\bar{l}(k)}(\mu;Y)$ and $\Upsilon^{k+1}(\mu;Y)$ strictly increase in μ and the former has a larger slope in μ, there is a unique μ' satisfying the conditions.

Third, there is a unique μ'' satisfying $U^{k+1,\bar{l}(k)+1}(\mu;Y) = \Upsilon^{k+1}(\mu;Y)$ and $0 < \mu' < \mu^*_{k,\bar{l}(k)}$. To see this, notice that if $\mu \ge \mu^*_{k,\bar{l}(k)} > \underline{\mu}_k$, we have $U^{k+1,\bar{l}(k)+1}(\mu;Y) > \Upsilon^{k+1}(\mu;Y)$ because $U^{k,\bar{l}(k)}(\mu;Y) \ge \Upsilon^k(\mu;Y)$. On the other hand, we have $U^{k+1,\bar{l}(k)+1}(0;Y) < \Upsilon^{k+1}(0;Y)$. Because both $U^{k+1,\bar{l}(k)+1}(\mu;Y)$ and $\Upsilon^{k+1}(\mu;Y)$ strictly increase in μ and the former has a larger slope in μ, there is a unique μ'' satisfying the conditions.

Finally, if $\mu'' < \mu_{k+1,\bar{l}(k)}$, define $\bar{l}(k+1) = \bar{l}(k)+1$ and $\mu^*_{k+1,\bar{l}(k+1)} = \mu''$, and define $\mu^*_{k+1,j} = \mu_{k+1,j}$ for $1 \le j \le \bar{l}(k)$. If $\mu'' \ge \mu_{k+1,\bar{l}(k)}$, define $\bar{l}(k+1) = \bar{l}(k)$ and $\mu^*_{k+1,\bar{l}(k+1)} = \mu'$, and define $\mu^*_{k+1,j} = \mu_{k+1,j}$ for $1 \le j < \bar{l}(k)$. With this construction of the sequence, It can be verified that all the other properties in (a) and (b) hold. ■

Remark B3 *If the deadline T satisfies $T = k - 1 > 0$, an important feature is that, if the expert's initial belief is relatively large, he prefers to disclose all information before the deadline is reached. For instance, if $\mu_0 \ge \mu_1^*$, the expert discloses all Y_0 in period $t = 0$ no matter what* *the deadline is. Thus, we need to characterize the cut-off values and value functions that describe the expert's utilization of information for any initial belief. The systems derived in Lemma B1 and Lemma B3 provide a full description of the expert's behaviors, which will be shown in the equilibrium.* Also notice that the value $\overline{\mu}_k$ we introduced in the main analysis equals $\mu^*_{k,\bar{l}(k)}$ here.

Lemma B4 *For any deadline $T = k-1$ and $2 \leq k \leq \overline{k}+1$, there exists an equilibrium in which the expert's payoff is $U^{k,l}(\mu;Y)$ if $\mu \in [\mu^*_{k,l}, \mu^*_{k,l-1}]$ and $1 \leq l \leq \bar{l}(k)$, is $\Upsilon^k(\mu;Y)$ if $\mu \in [\underline{\mu}_k, \mu^*_{k,\bar{l}(k)}]$, and is Y if $\mu \leq \underline{\mu}_k$.*

Proof. Suppose now the game is in period t with belief $\mu_t = \mu$ and information $Y_t = Y$, and the number of periods feasible for information disclosure is k, where $2 \leq k \leq \overline{k}+1$. Let y be an amount of information disclosed in period t, and let x be an amount of information self-used in period t. Consider the strategy profile and belief updating system described as follows.

Case 1: $\mu \in [\mu^*_{k,l}, \mu^*_{k,l-1}]$ and $1 \leq l \leq \bar{l}(k)$.

On the equilibrium path. Starting from period t, the expert's information disclosure follows an l-period scheme. In the first period of an l-period scheme, the biased investor betrays with probability $\varphi(\mu; \max\{\mu, \mu^*_{k-1,l-1}\})$. After observing a success in period t, the expert's belief updates to $\max\{\mu, \mu^*_{k-1,l-1}\}$ in period $t+1$. In this case, the expert's payoff is $U^{k,l}(\mu;Y)$.

Off the equilibrium path. The biased investor's deviation in period t is omitted by the expert; that is, after observing a success, the expert continues to update her belief to $\max\{\mu, \mu^*_{k-1,l-1}\}$ in period $t+1$. Thus, only the expert's deviations need to be considered.

First, consider $y > Y/\mathbf{q}^{l-1}$. If $y \in (Y/\mathbf{q}^{j-1}, Y/\mathbf{q}^{j-2}]$ and $2 \leq j \leq l$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j-2})$.[19] If a success is observed in period t, (a) if $j = 2$, the expert discloses $Y - y$ in the next period, and (b) if $j > 2$, starting from the next period the Case 1-equilibrium with a $j-2$-period scheme is played with probability λ_j and the Case 1-equilibrium with a $j-1$-period scheme is played with probability $1-\lambda_j$,[20] in which λ_j satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_j \beta_I \frac{Y-y}{\mathbf{q}^{j-3}} + (1-\lambda_j)\beta_I \frac{Y_t - y}{\mathbf{q}^{j-2}}]$$

and makes the biased investor being indifferent between cooperating and betraying in period t.

Second, consider $y \in (Y/\mathbf{q}^l, Y/\mathbf{q}^{l-1})$. (a) If $\mu \geq \mu^*_{k-1,l-1}$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 1-equilibrium with an $l-1$-period scheme is played.

[19] Here we have $\mu^*_{k-1,j-2} > \mu^*_{k,l-1}$ because $\mu^*_{k-1,l-2} > \mu^*_{k,l-1}$.

[20] By "Case 1-equilibrium", we mean the equilibrium described in Case 1. Similarly, "Case 2-equilibrium" refers to the equilibrium described in Case 2. The difference between these two classes of equilibria is that, in the equilibrium described in Case 1 the deadline is never reached, whereas in the equilibrium described in Case 2 the deadline is reached based on a series of successes.

(b) If $\mu < \mu^*_{k-1,l-1}$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,l-1})$. After observing a success in period t, if $\bar{l}(k-1) > l-1$, starting from the next period the Case 1-equilibrium with an $l-1$-period scheme is played with probability λ_l and the Case 1-equilibrium with an l-period scheme is played with probability $1-\lambda_l$, in which λ_l satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_l \beta_I \frac{Y-y}{\mathbf{q}^{l-2}} + (1-\lambda_l)\beta_I \frac{Y-y}{\mathbf{q}^{l-1}}];$$

if $\bar{l}(k-1) = l-1$, starting from the next period the Case 1-equilibrium with an $l-1$-period scheme is played with probability λ'_l and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda'_l$, in which λ'_l satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda'_l \beta_I \frac{Y-y}{\mathbf{q}^{l-2}} + (1-\lambda'_l)\beta_I \frac{Y-y}{\mathbf{q}^{k-2}}].$$

Third, consider $y \le Y/\mathbf{q}^l$. (a) If $\mu \ge \mu^*_{k-1,l-1}$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 1-equilibrium with an $l-1$-period scheme is played. (b) If $\bar{l}(k-1) = l$ and $\mu \in [\mu^*_{k-1,l}, \mu^*_{k-1,l-1}]$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 1-equilibrium with an l-period scheme is played. (c) Consider $\mu \in [\mu^*_{k-1,j+1}, \mu^*_{k-1,j}]$ such that $l \le j \le \bar{l}(k-1)-1$. If $y \le Y/\mathbf{q}^{j+1}$, the biased investor cooperates for sure and starting from period $t+1$ the Case 1-equilibrium with a $j+1$-period scheme is played. If $y \in (Y/\mathbf{q}^{j+1}, Y/\mathbf{q}^j]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with a j-period scheme is played with probability λ_j and the Case 1-equilibrium with a $j+1$-period scheme is played with probability $1-\lambda_j$, in which λ_j satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_j \beta_I \frac{Y-y}{\mathbf{q}^{j-1}} + (1-\lambda_j)\beta_I \frac{Y-y}{\mathbf{q}^{j}}].$$

(d) Consider $\mu \in [\underline{\mu}_{k-1}, \mu^*_{k-1,\bar{l}(k-1)}]$. If $y \le Y/\mathbf{q}^k$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 2-equilibrium with a $k-1$-period scheme is played. If $y \in (Y/\mathbf{q}^k, Y/\mathbf{q}^{\bar{l}(k-1)}]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,\bar{l}(k-1)})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with an $\bar{l}(k-1)$-period scheme is played with probability λ_k and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda_k$, in which λ_k satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda_k \beta_I \frac{Y-y}{\mathbf{q}^{\bar{l}(k-1)-1}} + (1-\lambda_k)\beta_I \frac{Y-y}{\mathbf{q}^{k-2}}].$$

Finally, if the expert self-uses x instead of disclosing y, starting from period $t+1$ the Case 1-equilibrium is played if $\mu \ge \mu^*_{k-1,\bar{l}(k-1)}$, the Case 2-equilibrium is played if $\mu \in [\underline{\mu}_{k-1}, \mu^*_{k-1,\bar{l}(k-1)})$, and $Y-y$ is self-used in period $t+1$ if $\mu < \underline{\mu}_{k-1}$.

Case 2: $\mu \in [\underline{\mu}_k, \mu^*_{k,\bar{l}(k)}]$.

On the equilibrium path. Starting from period t, the expert's information disclosure follows a k-period scheme. In the first period of a k-period scheme, the biased investor betrays with probability $\varphi(\mu; \max\{\mu, \underline{\mu}_{k-1}\})$. After observing a success in period t, the expert's belief updates to $\max\{\mu, \underline{\mu}_{k-1}\}$ in period $t+1$. In this case, the expert's payoff is $\Upsilon^k(\mu; Y)$.

Off the equilibrium path. The biased investor's deviation is omitted by the expert. So we only need to consider the expert's deviations.

First, consider $y > Y/\mathbf{q}^{k-1}$. (a) If $y \in (Y/\mathbf{q}^{k-1}, Y/\mathbf{q}^{\bar{l}(k-1)}]$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,\bar{l}(k-1)})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with an $\bar{l}(k-1)$-period scheme is played with probability λ'_k and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda'_k$, in which λ'_k satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda'_k \beta_I \frac{Y-y}{\mathbf{q}^{\bar{l}(k-1)-1}} + (1-\lambda'_k)\beta_I \frac{Y-y}{\mathbf{q}^{k-2}}].$$

(b) If $y \in (Y/\mathbf{q}^j, Y/\mathbf{q}^{j-1}]$ such that $2 \leq j \leq \bar{l}(k-1)$, the biased investor betrays with probability $\varphi(\mu; \mu^*_{k-1,j-1})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with a $j-1$-period scheme is played with probability λ'_j and the Case 1-equilibrium with a j-period scheme is played with probability $1-\lambda'_j$, in which λ'_j satisfies

$$\beta_I y = \alpha_I y + \delta[\lambda'_j \beta_I \frac{Y-y}{\mathbf{q}^{j-2}} + (1-\lambda'_j)\beta_I \frac{Y-y}{\mathbf{q}^{j-1}}].$$

(c) If $y > Y/(1+q)$, the biased investor betrays for sure. After observing a success, the expert discloses $Y-y$ in period $t+1$.

Second, consider $y < Y/\mathbf{q}^{k-1}$. (a) If $\mu \geq \underline{\mu}_{k-1}$, the biased investor cooperates for sure. Starting from period $t+1$ the Case 2-equilibrium with a $k-1$-period scheme is played. (b) If $\mu < \underline{\mu}_{k-1}$, the biased investor betrays with probability $\varphi(\mu; \underline{\mu}_{k-1})$. After observing a success in period t, starting from the next period the Case 2-equilibrium with a $k-1$-period scheme is played with probability λ''_k and the expert self-uses all $Y-y$ in period $t+1$ with probability $1-\lambda''_k$, in which λ''_k satisfies

$$\beta_I y = \alpha_I y + \delta\lambda''_k \beta_I \frac{Y-y}{\mathbf{q}^{k-2}}.$$

Finally, if the expert self-uses x instead of disclosing y, starting from period $t+1$ the Case 2-equilibrium is played if $\mu \in [\underline{\mu}_{k-1}, \mu^*_{k-1,\bar{l}(k)})$, and $Y-y$ is self-used in period $t+1$ if $\mu < \underline{\mu}_{k-1}$.

Case 3: $\mu \leq \underline{\mu}_k$.

On the equilibrium path. The expert self-uses $x = Y$ and her payoff is Y.

Off the equilibrium path. Consider the expert's deviations. (a) If $y \leq Y/\mathbf{q}^{k-1}$, the biased investor betrays with probability $\varphi(\mu;\underline{\mu}_{k-1})$. After observing a success in period t, starting from the next period the Case 2-equilibrium with a $k-1$-period scheme is played with probability λ_k'' and the expert self-uses all $Y-y$ in period $t+1$ with probability $1-\lambda_k''$, in which λ_k'' is denoted in the previous case. (b) If $y \in (Y/\mathbf{q}^{k-1}, Y/\mathbf{q}^{\bar{l}(k-1)}]$, the biased investor betrays with probability $\varphi(\mu;\mu^*_{k-1,\bar{l}(k-1)})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with an $\bar{l}(k-1)$-period scheme is played with probability λ_k' and the Case 2-equilibrium with a $k-1$-period scheme is played with probability $1-\lambda_k'$, in which λ_k' is denoted in the previous case. (c) If $y \in (Y/\mathbf{q}^j, Y/\mathbf{q}^{j-1}]$ such that $2 \leq j \leq \bar{l}(k-1)$, the biased investor betrays with probability $\varphi(\mu;\mu^*_{k-1,j-1})$. After observing a success in period t, starting from the next period the Case 1-equilibrium with a $j-1$-period scheme is played with probability λ_j' and the Case 1-equilibrium with a j-period scheme is played with probability $1-\lambda_j'$, in which λ_j' is denoted in the previous case. (d) If $y > Y/(1+q)$, then the biased investor betrays for sure. After observing a success, the expert discloses $Y-y$ in period $t+1$. (e) If the expert self-uses $x < Y$, in the next period she self-uses $Y-x$. ■

Remark B4 *The verification that the strategy profile and belief updating system consist of an equilibrium is similar to the one shown for Proposition 1. So we omit the details. The key property is that, given the biased investor's strategy, if* $\mu \in [\mu^*_{k,l}, \mu^*_{k,l-1}]$ *and* $1 \leq l \leq \bar{l}(k)$*, the expert prefers a* l*-period scheme to any other scheme or self-use; if* $\mu \in [\underline{\mu}_k, \mu^*_{k,\bar{l}(k)}]$*, the expert prefers a* k*-period scheme to any other scheme or self-use; and if* $\mu \leq \underline{\mu}_k$*, the expert prefers self-use to any scheme of information disclosure.*

Lemma B5 *If* $T = k-1$*,* $\mu \in [\mu^*_k, \mu^*_{k-1})$ *and* $2 \leq k \leq \overline{k}+1$*, there exists an equilibrium in which the expert's payoff is strictly larger than* $V^k(\mu;Y)$*.*

Proof. First, by a previous lemma, we have $\Upsilon^k(\mu;Y) > V^k(\mu;Y)$ if $\mu \in [\mu^*_k, \mu^*_{k-1})$ and $2 \leq k \leq \overline{k}+1$. Second, because $\mu^*_k > \underline{\mu}_k$, the expert's payoff is at least $\Upsilon^k(\mu;Y)$ in the equilibrium we have constructed. Thus, the statement is true. ■

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